Exam 3   Calc 3   12/1/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Give a parameterization, along with bounds, for the line segment from the origin to the point (2, –3, 5).

2. Compute div $\langle 3, 4x^2y, 2\,y^5 \cos x\rangle$.

3. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = 2x \cos y \, \mathbf{i} - x^2 \sin y \, \mathbf{j}$ and C is the line segment from (-3,0) to $(0,\pi)$.

4. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle 3y, 5x \rangle$ and C is the circle centered at the origin with radius 7, traversed clockwise.

5. Compute $\int_C \mathbf{G} \cdot d\mathbf{r}$ where $\mathbf{G}(x, y) = \langle 6xy, 2y^2 \rangle$ and C is the left half of a unit circle centered at the origin and traversed counterclockwise.

6. Show that curl $(\nabla f(x, y, z)) = \mathbf{0}$ provided that the mixed second-order partials of f are continuous. Make it clear how the requirement about the partials is involved.

7. Bunny is a Calc 3 student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod! I've never been so confused in my life! I though math was supposed to be about *numbers*, right? But now there's all this stuff with closed and stuff like that, and I missed the lecture about it because I had to get my hair cut. I thought it was things like doors and windows that could be closed, not *math*! It's totally unfair. I can't figure out how I'm supposed to know when something is closed or when it isn't, and I guess it's supposed to matter, right? So how do you tell if something's closed?"

   Explain clearly to Bunny what sorts of things are or aren't closed, and how you can tell.

8. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = x\,\mathbf{i} - z\,\mathbf{k}$ and S consists of the paraboloid $x = y^2 + z^2$ for $x \le 4$ and the disk $y^2 + z^2 \le 4$ for $x = 4$.

9. Evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = xyz\,\mathbf{i} + xy\,\mathbf{j} + x^2yz\,\mathbf{k}$, and S consists of the top and four sides (but not the bottom) of the cube with vertices $(\pm1, \pm1, \pm1)$, oriented outward.

10. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \dfrac{-y\,\mathbf{i} + x\,\mathbf{j}}{x^2 + y^2}$ and C consists of the right half of a unit circle centered at (1,0), then the line segment from (1,1) to (-1,1), then the left half of a unit circle centered at (-1,0), and then finally a line segment from (-1,-1) to (1,-1), all oriented counterclockwise.

Extra Credit (5 points possible):

Find the positively oriented simple closed curve C for which the value of the line integral $\int_C (y^3 - y)dx - 2x^3 dy$ is a maximum.