Exam 2 Real Analysis 1 11/10/2006

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. a) State the definition of continuity of a function at $x = a$.

   b) State the definition of continuity of a function on a set D.

2. a) State the definition of the derivative of a function at $x = a$.

   b) State the definition of differentiability of a function on a set D.

3. Give an example of a function that meets two of the three assumptions of Rolle’s Theorem but does not satisfy the conclusion of that theorem.

4. Give an example of a function $f : [a, b] \to \mathbb{R}$ whose range is an open and bounded interval.

5. State and prove the Difference Rule for derivatives.

6. State and prove the Boundedness Theorem.

7. State and prove the Mean Value Theorem.

8. Let E be an open subset of $\mathbb{R}$. Prove that $\mathbb{R} - E$ is closed.

9. Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ is differentiable on $\mathbb{R}$ and that $a, b \in \mathbb{R}$ with $a - b \geq 1$. Does there have to exist a $c \in \mathbb{R}$ for which $f(c) = \dfrac{f(a) + f(b)}{2}$? Why or why not?

10. Determine whether the function $f(x)=\begin{cases} x^2\sin\frac{1}{x} & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$ is differentiable when x=0.