Problem Set 4 Real Analysis Due 9/22/2006

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Prove or give a counterexample: If $\{a_n\}$ diverges then for any $M \in \mathbb{N}$, $\{a_n - M\}$ diverges also.

2. Prove or give a counterexample: If $\{a_n\}$ diverges and $\{b_n\}$ diverges, then it must be the case that $\{a_n + b_n\}$ diverges also.

3. Prove or give a counterexample: If $\{a_n\}$ diverges and $\{b_n\}$ converges, then it must be the case that $\{a_n + b_n\}$ diverges also.

4. Show that $\left\{\frac{2^n}{n!}\right\}$ converges.

5. Let $a, r \in \mathbb{R}$ with $r > 1$ and consider $\left\{\frac{a}{r^n}\right\}$. Prove that this sequence converges.

6. Prove that $\left\{\frac{2^n}{2^n+1}\right\}$ converges.

7. Show that a finite set has no accumulation points.

8. Show that the interval (0,1) has 1 as an accumulation point.

9. Give an example of a sequence where $\lim_{n\to\infty}\left|a_{n+1} - a_n\right| = 0$, but the sequence diverges.

10. Prove that if a_n and b_n are Cauchy sequences, then so is $\{a_n + b_n\}$.