Problem Set 5 Real Analysis Due 9/29/2006

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Let s_0 be an accumulation point of a set S. Prove that if every deleted neighborhood of s_0 contains at least one point of S, then every neighborhood of s_0 contains infinitely many points of S.

2. Let s_0 be an accumulation point of a set S. Prove that if every neighborhood of s_0 contains infinitely many points of S, then every deleted neighborhood of s_0 contains at least one point of S.

3. Prove that if $\{a_n\}$ is Cauchy and $\{a_n \mid n \in \mathbb{N}\}$ is finite, then $\{a_n\}$ is eventually constant.

4. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$. Prove (directly from the definition) that $\lim_{x\to\infty} [f(x) - g(x)] = A - B$.

5. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$. Prove (directly from the definition) that $\lim_{x\to\infty} [f(x) \cdot g(x)] = A \cdot B$.

6. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$. Prove (directly from the definition) that if $f(x) \le g(x)$, then $A \le B$.