Problem Set 6 Real Analysis Due 10/13/2006

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Give an example of functions f and g which are not continuous on $\mathbb{R}$, but where $f + g$ is.

2. Give an example of functions f and g which are not continuous on $\mathbb{R}$, but where $f \cdot g$ is.

3. Give an example of functions f and g which are not continuous on $\mathbb{R}$, but where $f \circ g$ is.

4. Let f be a function with domain D. Prove that if f is continuous at a, then there exists $\delta > 0$ for which f is bounded on $(a - \delta, a + \delta) \cap D$.

5. Let f be a function with domain D. Prove that if f is continuous at a and $f(a) > 0$, then there must exist a $\delta > 0$ for which $f(x) > \frac{1}{2} f(a)$ for all $x \in (a - \delta, a + \delta) \cap D$.

6. Give an example of a function which is defined on $\mathbb{R}$ but continuous at exactly one point.

7. Give an example of a function which is defined on $\mathbb{R}$ but continuous at exactly three points.

8. Let $f, g: D \to \mathbb{R}$ be functions which are continuous at a. Prove that the new function h defined by $h(x) = \max\{f(x), g(x)\}$ for all $x \in D$ is continuous at a.