Problem Set 9 Real Analysis Due 11/3/2006

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Give an example of an open cover for $\mathbb{R}$ for which no finite subcover exists, or show that one cannot exist.

2. Give an example of an open cover for $\{1/n \mid n \in \mathbb{N}\}$ for which no finite subcover exists, or show that one cannot exist.

3. Is the set $\{1/n \mid n \in \mathbb{N}\} \cup \{0\}$ compact? How do you know?

4. Prove or give a counterexample: If f and g are continuous on $\mathbb{R}$, then $f \cdot g$ is continuous on $\mathbb{R}$.

5. Prove or give a counterexample: If f and g are uniformly continuous on $\mathbb{R}$, then $f \cdot g$ is uniformly continuous on $\mathbb{R}$.

6. Prove or give a counterexample: If f,g are continuous on $\mathbb{R}$, satisfy $f(a) \leq g(a)$, and $f(b) \geq g(b)$, then there exists a $c \in [a,b]$ such that $f(c) = g(c)$.

7. Prove that the sum of n differentiable functions is differentiable.

8. Prove that the product of n differentiable functions is differentiable.

9. Prove that if $f(x) = x^n$ for some $n \in \mathbb{N}$, then $f'(x) = nx^{n-1}$.

10. Do problem 5.2.3 carefully.