Exam 3 Calc 2 11/2/2007

$x = r\cos\theta$
$y = r\sin\theta$

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Which of the following is (or are) a solution to the differential equation $\frac{dy}{dx} = 0.3y$?

$y' = 0.3y$

a) $y = 3e^{0.3t}$ $\quad y' = .9e^{.3t}$

b) $y = 5e^{0.3t}$ $\quad y' = 1.5e^{.3t}$

c) $y = e^{0.3t+7}$ $\quad y' = .3e^{.3t+7}$

a) $.9e^{.3t} \overset{?}{=} .3(3e^{.3t})$
$.9e^{.3t} = .9e^{.3t}$
yes

b) $1.5e^{.3t} \overset{?}{=} .3(5e^{.3t})$
$1.5e^{.3t} = 1.5e^{.3t}$
yes

c) $.3e^{.3t+7} \overset{?}{=} .3(e^{.3t+7})$
$.3e^{.3t+7} = .3e^{.3t+7}$
yes

Great

a, b, and c

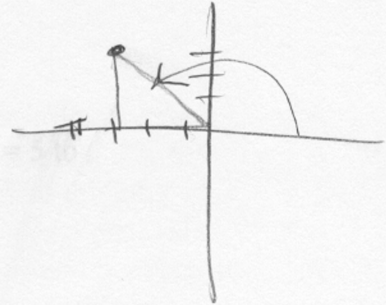
2. Set up an integral for the length of the cardioid with polar equation $r = 2 + \sin\theta$.

$$\text{Arc length} = \int \sqrt{r^2 + [r'(\theta)]^2}\, d\theta \qquad \frac{dr}{d\theta} = \cos\theta$$

$$= \boxed{\int_0^{2\pi} \sqrt{(2+\sin\theta)^2 + [\cos\theta]^2}\, d\theta}$$

Great

3. a) Convert the rectangular coordinates (–3, 3) to polar coordinates.



$x = r\cos\theta$ & $y = r\sin\theta$

$\left(3\sqrt{2}, \tfrac{3\pi}{4}\right)$

$3^2 + 3^2 = r^2$ $r = 3\sqrt{2}$

$9 + 9 = r^2$

$18 = r^2 \rightarrow r = \sqrt{18}$

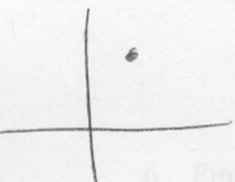
$\theta = \dfrac{3\pi}{4}$

10

b) Convert the polar coordinates (6, π/3) to rectangular coordinates.

$x = 6\cos(\pi/3)$ & $y = 6\sin(\pi/3)$

$x = 6 \cdot \frac{1}{2} = 3$ & $y = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$



$(3, 3\sqrt{3})$

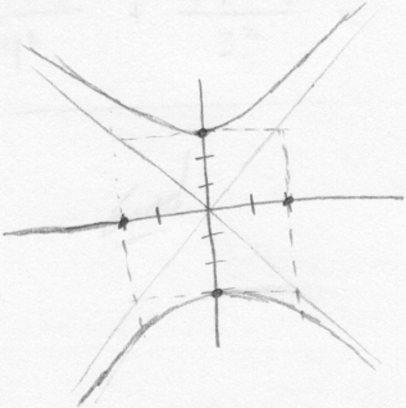
Excellent

4. Identify the conic section $4y^2 - 9x^2 = 36$ as an ellipse, hyperbola, or parabola, and sketch a rough graph.

$$\overset{\div 36}{4y^2 - 9x^2 = 36}\ \div 36$$

$$\frac{y^2}{9} - \frac{x^2}{4} = 1 \qquad \text{hyperbola}$$

$$\frac{y^2}{3^2} - \frac{x^2}{2^2} = 1$$



Excellent

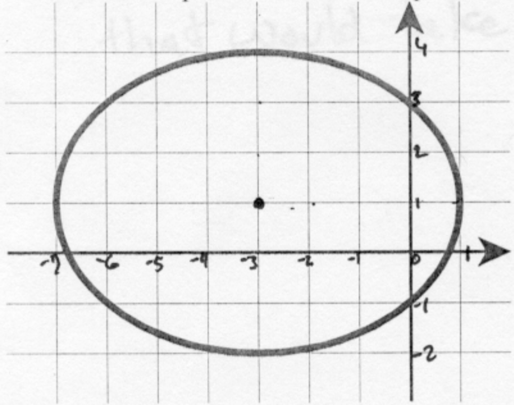
5. Let $\dfrac{dp}{dt} = 0.6p - 500$, and let $p(0) = 1000$. Use Euler's method with $\Delta t = 5$ to approximate $p(10)$ to the nearest hundred.

t	p	$\frac{dp}{dt}$	Δp
0	1000	100	500
5	1500	400	2000
10	3500		

Excellent

$\boxed{p(10) \approx 3500}$

6. Find an equation for the ellipse shown.



center $(-3, 1)$

$$\frac{(x+3)^2}{4^2} + \frac{(y-1)^2}{3^2} = 1$$

Good

$$\frac{(x+3)^2}{16} + \frac{(y-1)^2}{9} = 1$$

9. The differential equation $\frac{dp}{dt} = kp - h$ can be used to model populations subject to steady harvesting, as for example with Iowa's deer population, where k and h are constants representing the relative rate at which a population is reproducing and the rate at which that population is being harvested. Find a general solution to this differential equation.

constants

$$\frac{dP}{dt} = kp - h$$

$$\int \frac{1}{kp-h}\, dP = \int dt$$

$$\frac{1}{k} \ln|kp-h| = t + C$$

$$e^{\ln|kp-h|} = e^{k(t+C)}$$

$$|kp-h| = e^{k(t+C)}$$

$$kp - h = Ae^{kt}$$

$$kp = Ae^{kt} + h$$

$$\boxed{p = \frac{1}{k}\left(Ae^{kt} + h\right)}$$

$u = kp - h$

$du = k\, dp$

$\frac{1}{k} du = dp$

$\int \frac{1}{u} \cdot \frac{1}{k} du$

$\frac{1}{k} \int \frac{1}{u} du$

$\frac{1}{k} \ln|u|$

$e^{(kt + kC)}$

$e^{kt} \cdot \boxed{e^{kC}}$ constant

Ae^{kt}

can absorb absolute value bars

Nice Job!