Exam 1a   Calc 3   9/30/2008

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function $f(x, y)$ with respect to x.

2. Find an equation for the plane tangent to $z = 3x^2 + 2y^2 - 2y - 1$ at the point (3,5,66).

3. Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy - y^2}{x^2 + y^2}$ does not exist.

4. Write the appropriate version of the chain rule for $\dfrac{\partial u}{\partial t}$ in the case where $u = f(x, y, z)$, $x = x(s, t)$, $y = y(s, t)$, and $z = z(s, t)$. Make clear distinction between derivatives and partial derivatives.

5. Find the maximum and minimum values (and distinguish clearly which are which) of the function $f(x,y) = 4x + 6y$ subject to the constraint $x^2 + y^2 = 16$.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this Calc 3 stuff is killing me. It's like, geez, everything I had to memorize before in math is all a lie now, you know? Like on our first test, there was this question about what the graph of $y = x^2$ looked like, right? And obviously I memorized that before, so I picked the parabola graph, which was right there answer b in the multiple choices, right? But they said that one was wrong because of the z, which is crazy 'cause there's not even a z in the equation."

   Explain clearly to Biff why the graph of $y = x^2$ might not look like it used to.

8. Find and classify all critical points of the function $f(x, y) = 3xy - x^2y - xy^2$.

9. Show that the equation of the tangent plane to the hyperboloid $x^2/a^2 + y^2/b^2 - z^2/c^2 = 1$ at the point (x_0, y_0, z_0) can be written as $\frac{x\,x_0}{a^2} + \frac{y\,y_0}{b^2} - \frac{z\,z_0}{c^2} = 1$.

10. Jon has discovered an abandoned silver mine on family property in Arizona, and plans to resume mining operations as soon as possible (with the revenues going to a fund for enormous prize scholarships for the students earning the highest scores in Calculus 3 classes). However, it will first be necessary to construct a new access road. The mine is located at the point (4000, 2000, 4000) on Ideal Mountain, which has equation $z = \sqrt{6000^2 - x^2 - y^2}$. The road can have at most a 5% grade, meaning that as a truck moves along the road leaving the mine it cannot descend more that 5 feet for every 100 feet of horizontal travel. In which direction should the road leave the mine to descend as rapidly as feasible?

Extra Credit (5 points possible):
Give a parametric representation for the curve of intersection of the surfaces $z = x^2 + y^2/4$ and $z = x^2/4 + y^2$.