Exam 1b   Calc 3   9/30/2008

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function f(x, y) with respect to y.

2. Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ does not exist.

3. Find the directional derivative of the function $g(x, y) = \ln(x^2 + y^2)$ at the point $(-2, 1)$ in the direction of the vector $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$.

4. Write the appropriate version of the chain rule for $\dfrac{\partial u}{\partial t}$ in the case where $u = f(x, y)$, $x = x(r, s, t)$, and $y = y(r, s, t)$. Make clear distinction between derivatives and partial derivatives.

5. Let $f(x,y) = x^3 + 3xy^2$. Find the direction in which f is increasing fastest at the point $(-3, 1)$, and the rate of increase in that direction.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Find the maximum and minimum values (and distinguish clearly which are which) of the function $f(x,y) = x^2 - 4x + y^2$ subject to the constraint $x^2 + y^2 = 9$.

8. Biff is a student taking calculus at Enormous State University and he's a bit confused. Biff says "Man, Calc 3 is killing me. This Lagraze stuff makes no sense at all. I'm good with solving the equations and everything, and I got the right points on the test question, but I did the second derivative test and it said they were all maxes, and so I picked that answer from the crazy multiple choices, and when I got the test back it was wrong. I doublechecked it, and the guy who sits next to me did the same thing, so we're pretty sure they got the answer key wrong, but the professor won't admit it and just says we need to read the hypotheses of the theorem, whatever that means!"

   Help Biff out by explaining clearly why the second derivative test will or won't help him here, and how he should proceed.

9. Find and classify all critical points of the function $f(x,y) = x^3 - y^3 - 2xy + 6$.

10. Describe well the intersection of a sphere with radius 1 centered at the origin with a sphere of radius r centered at the point $(0,0,c)$.

Extra Credit (5 points possible):

We know that for continuous functions $f_{xy} = f_{yx}$. Are there functions for which $f_{xx} = f_{yy}$, but without f_{xx} or f_{yy} being zero? Either characterize such functions or explain why it couldn't happen.