

Exam 2a Calc 3 10/31/2008

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Set up a double integral for the volume of the solid lying under the elliptic paraboloid $x^2/4 + y^2/9 + z = 1$ and above the triangle $R = [-1, 1] \times [-2, 2]$.

2. Set up a triple integral for the first-octant volume below $z = xy$ inside a cylinder with radius 5 centered on the z -axis.

5. Suppose that a lamp has a bulb with a mean lifetime $\mu = 1000$ hours, which can be modeled with an exponential density function, and that as soon as one bulb burns out a second bulb replaces it. Set up an iterated integral for the probability that both bulbs burn out within a total of 1000 hours.

6. Find the Jacobian for the transformation $x = uv$, $y = vw$, $z = uw$.

7. Biff is a calculus student from Enormous State University, and he has a question. Biff says “So, these double integrals are killin’ me. On our quiz there was this one where I got it wrong and the TA said something about how I did it too much, like I did it for a rectangle, but it was supposed to be for a triangle. So then on the exam, I divided my answers by two, ‘cause a triangle is half of a rectangle, right? But then they marked those wrong too. So what’s up with that?”

Explain to Biff why his plan to divide by two for triangular regions does or doesn’t work.

8. Evaluate $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} 5 dz dy dx$.

9. The planet Mars is roughly spherical with a radius around 3380km. Suppose that Mars at one time had a polar ice cap a uniform 1km thick covering the region within 15° of its north pole (i.e., extending from the surface upwards). Set up iterated integrals for the z coordinate of the center of mass of that ice cap.

10. Set up an iterated integral (or integrals) for the volume within the rectangle $[0,1] \times [0,2]$ bounded between $z = x + y^2$ and $z = 1$.

Extra Credit [up to 5 points possible]:

Calculate the integral of $\cos z$ inside the surface $x^2 + y^2 + z^2 = 1$.