Exam 3a Calculus 3 11/25/2008

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Parametrize and give limits for a sphere with radius 2.

2. Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$, where $\mathbf{G}(x, y) = \langle xy^2, x^2y \rangle$ and C is a line segment from (2,1) to (5,–4).

3. Evaluate $\int_C x^4 dx + xy dy$, where C is the triangular curve consisting of the line segments from (0,0) to (1,0), from (1,0) to (0,1), and from (0,1) to (0,0).

4. Set up an integral for the surface area of the helicoid with vector equation $\mathbf{r}(u, v) = u \cos v\ \mathbf{i} + u \sin v\ \mathbf{j} + v\ \mathbf{k}$, for $0 \le u \le 1$, $0 \le v \le$ p.

5. Set up an integral for $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle x^4, xy - y^2 \rangle$ and C is the top half of a circle with radius 2, centered at the origin and traversed counterclockwise.

6. Show that for any function f(x,y,z) with continuous second partials, curl(grad f) = **0**. Make clear how you use the continuity condition.

7. Biff says “Okay, so I got it figured out. Calculus isn’t really math, dude. Math is when, like there’s numbers and you work out an answer. On our test, though, the last question was about, like, if you know a vector field is conservative, then what can you say about the line integrals on two legs of a right triangle and the line integral on the hypotenuse. That’s obviously, like, a philosophy question or something, not a math question, ‘cause it’s totally asking for an opinion!”

   Explain (clearly enough for Biff to understand) what sort of non-philosophical answer might be given to such a question.

8. Set up an integral for $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle y, z, x \rangle$ and S is the portion of the paraboloid $x = y^2 + z^2$ with $x \leq 9$, with outward orientation.

9. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is a sphere centered at the origin with radius 3 and outward orientation and $\mathbf{F}(x, y, z) = \langle xz, -z, y \rangle$.

10. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the disc of radius 3 centered at the origin in the xy-plane with downward orientation and $\mathbf{F}(x, y, z) = \langle xz, -z, y\rangle$.

Extra Credit (5 points possible): A vector potential for a vector field **F** is a vector field **A** for which curl **A** = **F**. Determine whether a vector potential exists for the vector field

$$\mathbf{F}(x, y, z) = \langle 2y - 1, 3z^2, 2xy\rangle$$

and if possible find one.