Exam 3b Calculus 3 11/25/2008

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Let $\mathbf{F}(x, y, z) = \langle 3x, y^3, z^5 \rangle$. Compute curl $\mathbf{F}$.

2. Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$, where $\mathbf{G}(x, y) = \langle xy^2, x^2y \rangle$ and C is a line segment from (2,1) to (5,–4).

3. Evaluate $\int_C x^4 dx + xy dy$, where C is the triangular curve consisting of the line segments from (0,0) to (1,0), from (1,0) to (0,1), and from (0,1) to (0,0).

4. Set up an integral for the surface area of the helicoid with vector equation $\mathbf{r}(u, v) = u \cos v \, \mathbf{i} + u \sin v \, \mathbf{j} + v \, \mathbf{k}$, for $0 \le u \le 1$, $0 \le v \le$ p.

5. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle xy^3, 2y^4 \rangle$ and C is the first-quadrant arc of a circle with radius 2, traversed counterclockwise.

6. Show that for any function f(x,y,z) with continuous second partials, curl(grad f) = $\mathbf{0}$. Make clear how you use the continuity condition.

7. Biff says "Okay, so I got it figured out. Calculus isn't really math, dude. Math is when, like there's numbers and you work out an answer. On our test, though, the last question was about, like, if you know a vector field is conservative, then what can you say about the line integrals on two legs of a right triangle and the line integral on the hypotenuse. That's obviously, like, a philosophy question or something, not a math question, 'cause it's totally asking for an opinion!"

   Explain (clearly enough for Biff to understand) what sort of non-philosophical answer might be given to such a question.

8. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is a sphere centered at the origin with radius 3 and outward orientation and $\mathbf{F}(x, y, z) = \langle xz, -z, y \rangle$.

9. Let $\mathbf{F}(x, y, z) = \langle xz, -z, y \rangle$. Set up an integral for $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the portion with $x \geq 0$ of a sphere centered at the origin with radius 3 and outward orientation.

10. Let $\mathbf{F}(x, y) = \langle 3x, x^2 - 5y \rangle$. For which circular paths C in the xy-plane will $\int_C \mathbf{F} \cdot d\mathbf{r}$ be positive?

Extra Credit (5 points possible): Find the positively oriented simple closed curve C for which the value of the line integral $\int_C \left(y^3 - y\right) dx - 2x^3 dy$ is a maximum.