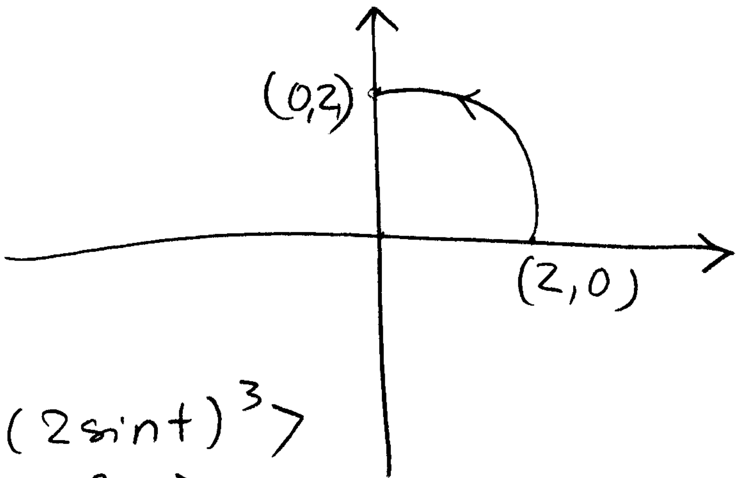
Quiz 4a Calculus 3 11/10/2008

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle xy^2, 2y^3 \rangle$ and C is the first-quadrant portion of a circle with radius 2, centered at the origin and traversed counterclockwise.



$x = 2\cos t$ $\quad 0 \leq t \leq \frac{\pi}{2}$

$y = 2\sin t$

$\vec{r}(2\cos t, 2\sin t)$

$\vec{r}'(-2\sin t, 2\cos t)$

$F(t) = \langle 2\cos t(2\sin t)^2, 2(2\sin t)^3 \rangle$

$= \langle 8\cos t \sin^2 t, 16\sin^3 t \rangle$

$\int_C F \cdot dr = \int_{t_1}^{t_2} F(t) \cdot \vec{r}' \, dt$

$= \int_0^{\pi/2} \langle 8\cos t \sin^2 t, 16\sin^3 t \rangle \cdot \langle -2\sin t, 2\cos t \rangle \, dt$

$= \int_0^{\pi/2} (-16\cos t \sin^3 t + 32\sin^3 t \cos t) \, dt$

$= (-4\sin^4 t + 8\sin^4 t)\Big|_0^{\pi/2} = 4\sin^4 t \Big|_0^{\pi/2}$

$= 4 \cdot \sin^4 \frac{\pi}{2}$

$= 4 \cdot (1) = \underline{4}$

Great

2. Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$, where $\mathbf{G}(x, y) = \langle xy^2, x^2y \rangle$ and C is a line segment from (2,1) to (5,–4).

$g(x,y) = \frac{1}{2}x^2y^2$ is a potential function, so

$$\int_C \vec{G} \cdot d\vec{r} = \frac{1}{2}x^2y^2 \Big|_{(2,1)}^{(5,-4)}$$ by the Fun. Theorem for Line Integrals

$$= \frac{1}{2}(5)^2(-4)^2 - \frac{1}{2}(2)^2(1)^2$$

$$= 200 - 2$$

$$= 198$$