Quiz 4b **Calculus 3** **11/10/2008**

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$, where $\mathbf{G}(x, y) = \langle xy^2, x^2y \rangle$ and C is a line segment from (2,1) to (5,–4).

potential function $\rightarrow$ $g(x,y) = \dfrac{x^2y^2}{2}$

Fundametal theorem:

$\int_C \vec{G} \cdot dr = g(\vec{r}(b)) - g(\vec{r}(a))$ $b=(5,-4)$ $a=(2,1)$

$g(5,-4) = 200$

$g(2,1) = 2$

$= g(5,-4) - g(2,1)$

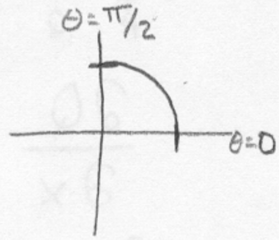
$= 200 - 2 = \boxed{198}$

Great.

2. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle xy^3, 2y^4\rangle$ and C is the first-quadrant portion of a circle with radius 1, centered at the origin and traversed counterclockwise.

$\frac{\partial P}{\partial y} = 3xy^2$ So do the long way!

$\frac{\partial Q}{\partial x} = 0$



$\vec{r}(t) = \langle \cos t, \sin t\rangle$ for $0 \le t \le \frac{\pi}{2}$

$\vec{F}(\vec{r}(t)) = \langle \cos t \sin^3 t, 2\sin^4 t\rangle$

$\vec{r}\,'(t) = \langle -\sin t, \cos t\rangle$

$$\int_0^{\pi/2} \langle \cos t \sin^3 t, 2\sin^4 t\rangle \cdot \langle -\sin t, \cos t\rangle \, dt$$

$$\int_0^{\pi/2} (-\cos t \sin^4 t + 2\cos t \sin^4 t)\, dt$$

$$\int_0^{\pi/2} (\cos t \sin^4 t)\, dt$$

Nice Work!

U-substitution

let $u = \sin t$

$\frac{du}{dt} = \cos t$

$\frac{du}{\cos t} = dt$

$$\int_{t=0}^{t=\pi/2} \cancel{\cos t}\, u^4 \frac{du}{\cancel{\cos t}}$$

$$\int_{t=0}^{t=\pi/2} u^4 \, du$$

$$\frac{1}{5}u^5 \Big|_{t=0}^{t=\pi/2} = \frac{1}{5}\sin^5 t \Big|_0^{\pi/2} = \frac{1}{5}\sin^5\left(\frac{\pi}{2}\right) - \frac{1}{5}\sin^5(0)$$

$$\frac{1}{5}(1) \quad - \quad 0 \quad = \boxed{\frac{1}{5}}$$