Exam 2   Real Analysis 1   11/10/2008

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of compactness.

2. State the (local) definition of continuity.

3. Give an example of a function $f:\mathbb{R} \to \mathbb{R}$ that fails to be differentiable at exactly one point.

4. State the Intermediate Value Theorem.

5. State and prove the Quotient Rule for Derivatives.

6. Prove directly from the definition that $f(x) = 5x + 2$ is continuous at $x = 3$.

7. State and prove the Mean Value Theorem.

8. Prove that the complement of a closed subset of $\mathbb{R}$ is open.

9. Is the function $f(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}$ a rational function? Support your answer.

10. Prove or give a counterexample: A function f which is differentiable at $x = a$ must be differentiable on an interval of the form $(a - \varepsilon, a + \varepsilon)$ for some $\varepsilon > 0$.