Problem Set 3 Real Analysis Due 9/14/2008

Several of these problems will be graded, with each graded problem worth 5 points. Clear, complete, and legible justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Do Exercise 2.1.9.

2. We call a sequence *eventually bounded* iff there exists some $n^* \in \mathbb{R}$ and $M \in \mathbb{R}$ such that for all $n > n^*$, we have $|a_n| < M$. Prove that any eventually bounded sequence is bounded.

3. Prove or give a counterexample: Any eventually bounded sequence converges.

4. If $\{a_n\}$ converges to A and there exists n_1 such that $a_n > 0$, for all $n > n_1$, prove that $A \geq 0$ without using Theorem 2.2.1.

5. Prove or give a counterexample: If $\{a_n\}$ converges to 0 and $\{b_n\}$ is another sequence, then $\{a_n b_n\}$ converges to 0 as well.

6. Prove or give a counterexample: If $\{a_n\}$ diverges to $-\infty$ then for any $M \in \mathbb{N}$, $\{a_n + M\}$ diverges to $-\infty$ also.

7. Prove or give a counterexample: If $\{a_n\}$ and $\{b_n\}$ oscillate, then $\{a_n + b_n\}$ oscillates also.

8. Prove or give a counterexample: If $\{a_n\}$ diverges to $+\infty$ and $\{b_n\}$ converges, then it must be the case that $\{a_n + b_n\}$ diverges also.