Exam 3 Calc 2 11/6/2009

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. a) Convert the point with rectangular coordinates $(3,-3)$ to polar coordinates.

   b) Convert the point with polar coordinates $(2, \pi/2)$ to rectangular coordinates.

2. Determine whether $y = x\, e^{x}$ is a solution to the differential equation $y'' - 2y' + y = 0$.

3. Consider the curve with parametric equations $x = 10 - t^2$, $y = t^3 - 12t$. Find the coordinates of all points on this curve where the tangent line is horizontal.

4. Set up an integral for the length of the curve with parametric equations $x = \cos t$, $y = \sin 2t$.

5. Find the median of the p.d.f. $p(x) = \begin{cases} \frac{x}{2} & \text{if } 0 \le x \le 2 \\ 0 & \text{if } x<0 \text{ or } x>2 \end{cases}$.

6. Write the conic section with equation $16x^2 + y^2 - 8y = 0$ in standard form, identify it as a parabola, hyperbola, or ellipse, and give coordinates of its vertices.

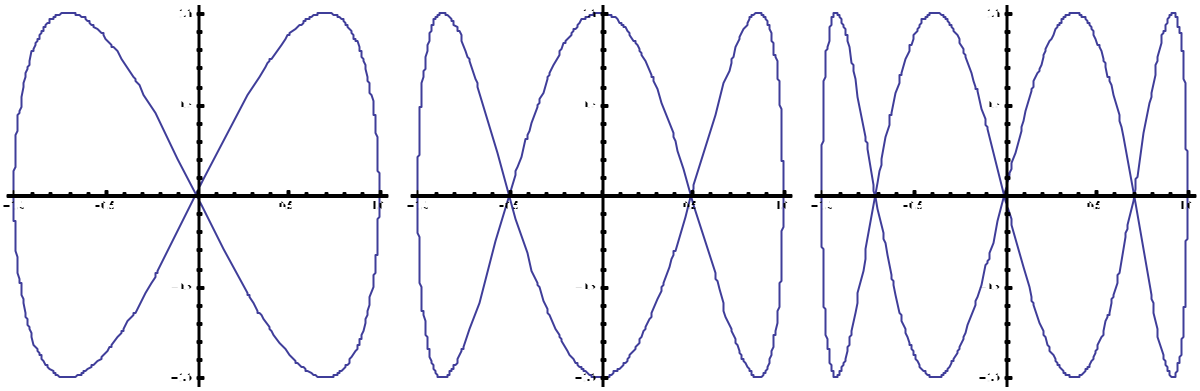
7. Bunny is a calculus student at Enormous State University, and she has a question. Bunny says "Ohmygod, this is so hard. Our TA gave us, like, this hint? And it was that we needed to know something about the error if you use the Oiler's method, like for approximating differential equators, you know? But I looked in the book and there aren't any formulas for it at all. I remember with stuff like the midpoint approximations there were formulas, and I could do that, but now I'm totally panicking!"

   Help Bunny by explaining what's most important to know about the errors involved in using Euler's method.

8. The buildup of mulch on a forest floor can be modeled by the differential equation $\frac{dm}{dt} = 2 - 0.4m$, (where m is the amount of mulch at time t, 2 inches of new material falls to the forest floor each year, and 40% of the material present decays continuously). Find a solution to this differential equation satisfying the initial condition $m(0) = 0$.

9. Find the area inside both $r = 1 + \cos\theta$ and $r = 1 - \sin\theta$.

10. Jon is fascinated by curves with parametric equations $x = \cos t$, $y = \sin n\,t$, where n is a natural number. He's decided to call them n-finity curves. Set up integrals for the areas inside of n-finity curves based on the value of n [2-finity, 3-finity, and 4-finity curves are shown below].



Extra Credit (5 points possible):
Evaluate the integrals from problem 10.
[The identities $\sin 2\theta = 2\sin\theta\cos\theta$ and $\sin(x + y) = \sin x\cos y + \cos x\sin y$ might be helpful.]