Exam 4 Calc 2 12/4/2009

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find the sum of the series $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\ldots$.

2. Determine whether the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges or diverges.

3. a) Write the MacLaurin polynomial of degree 6 for $\cos x$.

b) Determine whether the series $\sum_{n=0}^{\infty} \cos n$ converges or diverges.

4. Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n^2 - 3}$ converges or diverges.

5. Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ converges or diverges.

6. Determine whether the series $\sum_{n=0}^{\infty} \frac{1}{1+n!}$ converges or diverges.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, I think they just try really hard to make this series stuff more confusing than it has to be. Like, they tell you twenty different tests when pretty much you only ever use the ratio thing anyway, right? And also they say the ratio thing doesn't tell you anything when it gives a one, right, but I'm pretty sure really those always converge. They just try to confuse us because otherwise everybody would get in on those high-paying math jobs, right?"

   Help Biff by addressing his concerns as clearly as possible.

8. Determine the radius of convergence of the power series $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$.

9. Determine whether 1 and –1 are included in the interval of convergence of the power series

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}.$$

10. a) Use a polynomial of degree 4 to approximate $\int_0^{0.1} e^{\left(-x^2\right)} dx$.

b) What can you say about how accurate your approximation in part a is, and why?

Extra Credit (5 points possible):

Use a polynomial approximation to estimate $\sqrt[3]{6}$ well.

Reference Sheet: Series Tests Calculus 2 11/17/2009

The Geometric Series Test: If a series is of the form $\sum_{n=1}^{\infty} a \cdot r^{n-1}$, then the series converges $\left(\text{to } \dfrac{a}{1-r}\right)$ if and only if $|r| < 1$

The Integral Test: Suppose f(x) is a continuous, positive, decreasing function on [c, ∞) for some $c \geq 0$, with $a_n = f(n)$ for all n,

- If $\int_c^{\infty} f(x)\,dx$ converges, then Σa_n converges also.
- If $\int_c^{\infty} f(x)\,dx$ diverges, then Σa_n diverges also.

The Comparison Test: If Σa_n and Σb_n are both series with their terms all positive, and

- $a_n \leq b_n$ with Σb_n convergent, then Σa_n converges also.
- $a_n \geq b_n$ with Σb_n divergent, then Σa_n diverges also.

The Limit Comparison Test: If Σa_n and Σb_n are both series with their terms all positive, and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

for some finite, positive number L, then either both series converge or both series diverge.

The Alternating Series Test: If Σa_n is a series for which

- the signs alternate, i.e. a_n and a_{n+1} have opposite signs for all n
- the sequence $\{|a_n|\}$ tends to zero, i.e. $\lim_{n\to\infty} |a_n| = 0$
- the sequence $\{|a_n|\}$ is decreasing, i.e. $|a_{n+1}| \leq |a_n|$ for all n

then the series converges.

The Ratio Test: If Σa_n is a series for which

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$$

then

- if $L < 1$ then the series converges absolutely.
- if $L > 1$ (or if the limit diverges to $+\infty$) then the series diverges.