

Each problem is worth 0 points. In the event of an actual quiz, you would have received warning.

1. Write an equation for the plane tangent to $z = x^3 - xy^2$ at $(-1, 2)$.

Well, $z_x = 3x^2 - y^2$ and $z_y = -2xy$,

so $z_x(-1, 2) = 3(-1)^2 - (2)^2 = -1$ and $z_y(-1, 2) = -2(-1)(2) = 4$

Also at $(-1, 2)$ we have $z = (-1)^3 - (-1)(2)^2 = 3$

So using our general form for a tangent plane

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

we have the equation

$$z - 3 = -1(x + 1) + 4(y - 2)$$

2. Write an equation for the plane tangent to $x^2 + y^2 + z^2 = 25$ at the point $(-3, 4, 0)$.

If we let $f(x, y, z) = x^2 + y^2 + z^2$, then this sphere is one level surface of the sphere. Computing the gradient of this function,

$$\nabla f(x, y, z) = \langle 2x, 2y, 2z \rangle$$

and at $(-3, 4, 0)$ we have

$$\nabla f(-3, 4, 0) = \langle 2(-3), 2(4), 2(0) \rangle = \langle -6, 8, 0 \rangle$$

Then since the gradient is perpendicular to the level surface, we can use this as our normal vector and use the

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

form of an equation for the plane to write

$$\langle -6, 8, 0 \rangle \cdot (\langle x, y, z \rangle - \langle -3, 4, 0 \rangle) = 0$$

or

$$-6(x + 3) + 8(y - 4) = 0$$