Quiz 7 Calculus 3 11/4/2009

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle xy^2, 2y^3\rangle$ and C is the first-quadrant portion of a circle with radius 3, centered at the origin and traversed counterclockwise.

I $x(t) = 3\cos t$

$y(t) = 3\sin t$

$\vec{r}(t) = \langle 3\cos t, 3\sin t\rangle$

$0 \le t \le \frac{\pi}{2}$

II $\vec{F}(\vec{r}(t)) = \langle 3\cos t(3\sin t)^2, 2(3\sin t)^3\rangle$

III $\vec{r}\,'(t) = \langle -3\sin t, 3\cos t\rangle$

IV $\int_0^{\pi/2} \langle 3\cos t\, 9\sin^2 t, 54\sin^3 t\rangle \cdot \langle -3\sin t, 3\cos t\rangle\, dt$

V $\int_0^{\pi/2} -81\cos t\sin^3 t + 162\sin^3 t\cos t\; dt$

$\int_0^{\pi/2} 81\cos t\sin^3 t\; dt = \int_0^{\pi/2} 81u^3\, du = \frac{81}{4}u^4\Big|_{t=0}^{t=\pi/2}$

$u = \sin t$

$du = \cos t\, dt$

$\frac{81}{4}\sin^4 t\Big|_0^{\pi/2}$

Nice Work!

$\frac{81}{4}\sin^4(\pi) - \frac{81}{4}\sin^4(0)$

$\boxed{\frac{81}{4}}$

2. Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$, where $\mathbf{G}(x, y) = \langle y + 2xy, x + x^2 \rangle$ and C is a line segment from $(-1,2)$ to $(3,1)$.

$g(x,y) = xy + x^2 y$ is a potential function for

$\vec{G}(x,y)$

So by the Fun Thrm of Line Integrals

$g(3,1) - g(-1,2) =$ the line integral

$3+9-(-2+2) = 12$

Great.