Quiz 7 **Calculus 3** **11/11/2009**

Each problem is worth 5 points. Clear and complete justification is required for full credit.

dot product

1. Let $\mathbf{F}(x,y,z)=\left\langle \frac{-z}{\sqrt{x^2+y^2+z^2}}, 2, \frac{x}{\sqrt{x^2+y^2+z^2}} \right\rangle$. Compute div $\mathbf{F}$.

$$\text{div } \vec{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \left\langle \frac{-z}{\sqrt{x^2+y^2+z^2}}, 2, \frac{x}{\sqrt{x^2+y^2+z^2}} \right\rangle$$

$$\text{div } \vec{F} = \frac{\partial}{\partial x}\left(\frac{-z}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial y}(2) + \frac{\partial}{\partial z}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\text{div } \vec{F} = \frac{xz}{(x^2+y^2+z^2)^{\frac{3}{2}}} + 0 + \frac{-zx}{(x^2+y^2+z^2)^{\frac{3}{2}}} = \boxed{0}$$

Excellent!

scalar

2. Let $\mathbf{G}(x, y, z) = xyz\,\mathbf{i} + z\,\mathbf{j} - 5y^2\,\mathbf{k}$. Compute curl $\mathbf{G}$.

$\vec{G} = \langle xyz, z, -5y^2 \rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xyz & z & -5y^2 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ z & -5y^2 \end{vmatrix} - \hat{j}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ xyz & -5y^2 \end{vmatrix} + \hat{k}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ xyz & z \end{vmatrix}$$

$$= (-10y - 1)\hat{i} - (0 - xy)\hat{j} + (0 - xz)\hat{k}$$

$$= \langle -10y - 1, xy, -xz \rangle$$

Nicely Done!