Exam 1 Calc 3 9/28/2010

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function f(x, y) with respect to x.

2. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy-y^2}{x^2+y^2}$ does not exist.

3. Write the appropriate version of the chain rule for $\dfrac{\partial u}{\partial t}$ in the case where $u = f(x, y)$, $x = x(s, t)$, and $y = y(s, t)$. Make clear distinction between derivatives and partial derivatives.

4. Let $g(x, y) = \cos\sqrt{x^2 + y^2}$. Find the directional derivative of g in the direction of $\langle 2, -1 \rangle$ at the point $\left({}^{p}\!/_{2}, 0 \right)$.

5. Let $f(x,y) = x/y^3$. Find the maximum rate of change of f at the point (2,3) and the direction in which it occurs.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this Calc 3 stuff is killing me. These two other kids in the class seem to know everything, which is totally unfair, so I think we should get graded on a curve, you know? But so they were saying something about how there was this way to do this one problem from the exam, where it was about maxes and mins and stuff, but they said you could do it with gradients. That's total crap, because it's in a different section of the book, you know? So what the heck do gradients have to do with maxes?"

   Explain clearly to Biff some significant connection between local optimization and gradients.

8. Find the maximum and minimum values of the function $f(x, y) = 3x^2 + y^2$ on the circle $x^2 + y^2 = 4$.

9. Describe the collection of points on the surface $z = x^2 + y^2$ for which the tangent plane passes through the point $(5, 0, 0)$.

10. Let $f(x, y) = x^4 - 5x^2 + y^2 + 2xy$. Find all critical points of this function and classify them as local maxima, local minima, or saddle points.

Extra Credit (5 points possible):
If we vary the function from problem 10 to be $f(x, y) = x^4 - \beta x^2 + y^2 + 2xy$, then for which values of β will the function have two local minima?