

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find the directional derivative of $f(x, y) = 1 + 2x\sqrt{y}$ at the point $(2, 4)$ in the direction of $\mathbf{v} = \langle -3, 4 \rangle$.

$$\begin{aligned} D_{\mathbf{u}} &= \langle f_x(x, y), f_y(x, y) \rangle \cdot \langle a, b \rangle \\ &= \langle 4, 1 \rangle \cdot \langle -3/5, 4/5 \rangle \\ &= -12/5 + 4/5 \\ &= \boxed{-8/5} \end{aligned}$$

Wonderful!

$$\begin{aligned} \text{unit vector} &= \frac{\langle -3, 4 \rangle}{|\langle -3, 4 \rangle|} \\ &= \frac{\langle -3, 4 \rangle}{\sqrt{(-3)^2 + (4)^2}} \\ &= \langle -3/5, 4/5 \rangle \end{aligned}$$

$$\begin{aligned} f_x(2, 4) &= 2\sqrt{y} = 2\sqrt{4} = 4 \\ f_y(2, 4) &= 2x(1/2)y^{-1/2} \\ &= \frac{2x}{\sqrt{y}} = 1 \end{aligned}$$

2. Find the gradient of $f(x, y) = y^2/x$ at $P(1, 2)$.

$$\text{gradient} = \langle f_x(x, y), f_y(x, y) \rangle$$

$$f_x(x, y) = -\frac{y^2}{x^2}$$

$$f_x(1, 2) = \frac{-4}{1} = -4$$

$$f_y(x, y) = \frac{2y}{x}$$

$$f_y(1, 2) = \frac{4}{1} = 4$$

$$\begin{aligned} &\langle f_x(1, 2), f_y(1, 2) \rangle \\ &= \langle -4, 4 \rangle \end{aligned}$$

Great