Quiz 5 **Calculus 3** **11/15/2010**

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle xy^2, 2y^3\rangle$ and C is the first-quadrant portion of a circle with radius 3, centered at the origin and traversed counterclockwise.

$$\left.\begin{aligned} x(t) &= 3\cos t \\ y(t) &= 3\sin t \end{aligned}\right\}\quad 0 \le t \le \pi/2$$

$$\vec{r}(t) = \langle 3\cos t, 3\sin t\rangle$$

$$\vec{F}(\vec{r}(t)) = \langle 3\cos t(3\sin t)^2, 2(3\sin t)^3\rangle$$

$$\vec{r}\,'(t) = \langle -3\sin t, 3\cos t\rangle$$

$$\int_0^{\frac{\pi}{2}} \langle 27\cos t\sin^2 t, 54\sin^3 t\rangle \cdot \langle -3\sin t, 3\cos t\rangle\, dt$$

$$\int_0^{\frac{\pi}{2}} -81\cos t\sin^3 t + 162\sin^3 t\cos t\; dt$$

u substitution...

$u = \sin t$

$du = \cos t$

$$\int_0^{\frac{\pi}{2}} 81\sin^3 t\cos t\; dt$$

Beautiful!

$$81\int_0^{\pi/2} u^3 \cdot du$$

$$81\left[\tfrac{1}{4}u^4\right]_0^{\pi/2} \rightarrow \frac{81}{4}\sin^4 t\Big]_0^{\pi/2} = \frac{81}{4}$$

2. Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$, where $\mathbf{G}(x, y) = \langle y + 3x^2y, x + x^3 \rangle$ and C is a line segment from $(-1,2)$ to $(3,1)$.

Is there a g such that $\vec{G} = \nabla g$?

$g_x = y + 3x^2y \qquad g_y = x + x^3$

$g_{xy} = 1 + 3x^2 \qquad g_{yx} = 1 + 3x^2$

yes.

$g = xy + x^3y + K$

So, we can apply the Fun. theorem of line integrals.

$\int_C \vec{G} \cdot d\vec{r} = \int_C \nabla g \cdot d\vec{r} = g(3,1) - g(-1,2)$

$= (3)(1) + (3)^3(1) + K - (-1)(2) - (-1)^3(2) - K$

Well done!

5

$= 3 + 27 + \cancel{K} + 2 + 2 - \cancel{K}$

$= \boxed{34}$