Exam 2 Real Analysis 1 11/11/2010

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of a closed subset of $\mathbb{R}$.

2. a) State the (local) definition of continuity.

   b) Give an example of a function that fails to be continuous at $x = 0$.

3. a) State the definition of a compact set.

b) State the Heine-Borel Theorem.

4. State the Boundedness Theorem.

5. State and prove the Quotient Rule for Derivatives.

6. State and prove Brouwer’s Fixed Point Theorem.

7. State and prove the Extreme Value Theorem.

8. State and prove Rolle’s Theorem.

9. State and prove Fermat’s Theorem.

10. The Racetrack Principle: Let f and g be differentiable functions from $[a, b]$ to $\mathbb{R}$, and suppose $f(a) = g(a)$. Show that if $f'(x) \geq g'(x)$ for all $x \in [a, b]$, then $f(x) \geq g(x)$ for all $x \in [a, b]$.