Exam 1b Calc 1 9/23/2011

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Use the graph of $g(x)$ at the bottom of the page for problems 1 through 3:

1. Find the following limits:
   a) $\lim_{x\to -3} g(x) = 2$
   b) $\lim_{x\to 0} g(x) = 2$
   c) $\lim_{x\to 2^-} g(x) = 2$
   d) $\lim_{x\to 2^+} g(x) = 1$
   e) $\lim_{x\to 2} g(x)$ "DNE"

Good.

2. For which values of x does the function fail to be continuous?

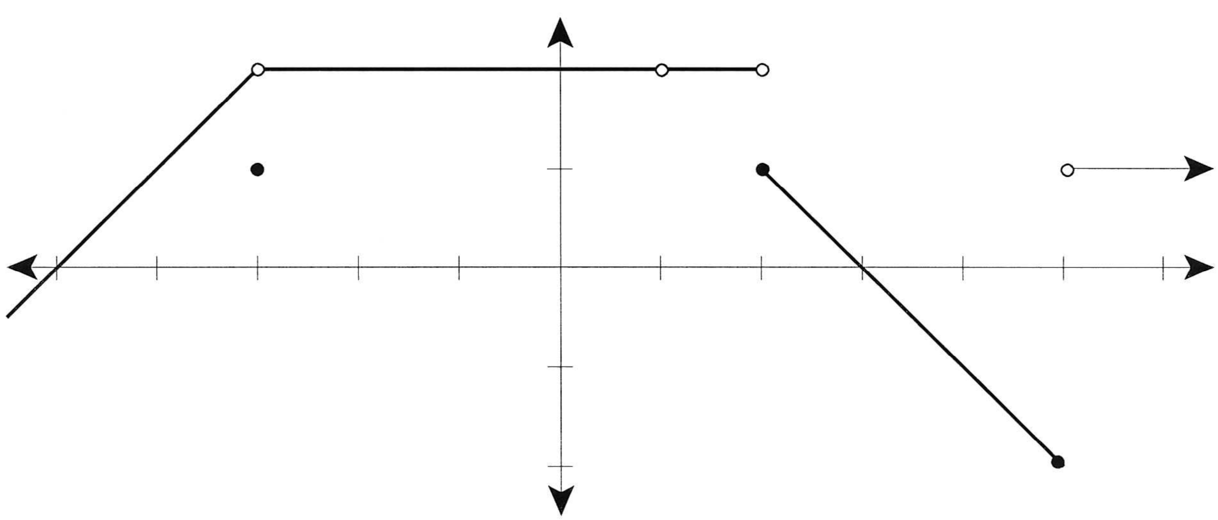
$x = -3, 1, 2, 5$

Good

3. Find a slope function for $g(x)$.

$$g(x) \begin{cases} 1 & x < -3 \\ 0 & -3 < x < 1,\ 1 < x < 2,\ x > 5 \\ -1 & 2 < x < 5 \end{cases}$$

Excellent!



4. Fill each blank below with a limit rule justifying that equality. [List of rules on last page]

$\swarrow$ Difference rule for limits

$$\lim_{x\to 2}\left(6x^2-5\right) = \lim_{x\to 2}\left(6x^2\right)-\lim_{x\to 2}5$$

$\swarrow$ constant multiple rule for limits

$$= 6\lim_{x\to 2}\left(x^2\right)-\lim_{x\to 2}5$$

$\swarrow$ Power rule for limits

$$= 6\left(\lim_{x\to 2}x\right)^2-\lim_{x\to 2}5$$

Great

$\swarrow$ Rule X for limits

$$= 6(2)^2-\lim_{x\to 2}5$$

$\swarrow$ Constant rule for limits

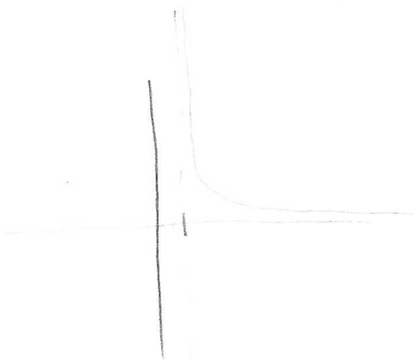
$$= 6\cdot 4-5$$

$$= 24-5$$

$$= 19$$

5. Evaluate $\lim_{x \to 1^-} \dfrac{3x^2}{x^2 - 2x + 1}$.

X	1	1.0001	1.001	1.01	1.1	2
f(x)		200,010,000	3,006,003	30603		12



$$\lim_{x \to 1^-} \frac{3x^2}{x^2-2x+1} = \underline{\infty}$$

Yes.

6. Evaluate $\lim_{x \to \infty} \dfrac{3x^2}{x^2 - 2x + 1}$.

$$\lim_{x \to \infty} \frac{\frac{3x^2}{x^2}}{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2}} = \frac{3}{1 - \underbrace{\cancel{2/x}}_{0} + \underbrace{\cancel{1/x^2}}_{0}}$$

$$= \frac{3}{1-0+0} = \frac{3}{1} = \boxed{3}$$

Nice!

7. Evaluate $\lim_{h \to 0} \dfrac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h \to 0} \dfrac{\frac{1}{3+h} \cdot \frac{3}{3} - \frac{1}{3} \cdot \frac{3+h}{3+h}}{h}$

$$= \lim_{h \to 0} \frac{\frac{3-(3+h)}{3(3+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{3(3+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-1}{9}$$

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. I did good in math in high school, but everything is totally different now. I mean, like, the one thing you knew before was that if there was a function with, like, a fraction, so like there was x minus 3 underneath, then there'd be one of those vertical asymptom things when x was 3, right? But we had this one on our test, and it was like $\dfrac{x^2-7x+6}{x^2-1}$, so it should have asymptoms when x is 1 and –1, right? But on our test I said that and they marked it wrong."

Help Biff by explaining as clearly as you can why the graph of his function does not have the vertical asymptote he expected when $x = +1$, and what is in fact going on near that x value.

Biff, try factoring your function first to see what's really going on.

$$f(x) = \frac{x^2-7x+6}{x^2-1} = \frac{(x-6)(x-1)}{(x+1)(x-1)}$$ Look! $x-1$ matches!

So when $x=-1$ the denominator is zero, and close to there the denominator is close to zero, making the fraction really big. But when $x=1$ it's a little different. Right at $x=1$ the function is undefined, but near there both numerator and denominator are close to zero in a way that cancels out. As long as x isn't actually 1 itself, you can cancel the $x-1$ factors, and the graph looks just like $\frac{x-6}{x+1}$.

9. Evaluate $\lim\limits_{x \to a} \dfrac{x^4-a^4}{x-a}$.

$$\Rightarrow \lim_{x \to a} \frac{(x^2-a^2)(x^2+a^2)}{x-a}$$

$$\Rightarrow \lim_{x \to a} \frac{\cancel{(x-a)}(x+a)(x^2+a^2)}{\cancel{x-a}}$$

Well Done.

$$\Rightarrow \lim_{x \to a} (x+a)(x^2+a^2) = \underline{4a^3}$$

10. a) Determine whether $f(x)=\dfrac{|x-2|(x+1)}{x-2}$ is continuous where $x=2$, and explain your reasoning clearly.

Absolute value switches sign for $x<2$

$$\lim_{x\to 2^-}\frac{|x-2|(x+1)}{x-2}=\lim_{x\to 2^-}\frac{-\cancel{(x-2)}(x+1)}{\cancel{x-2}}=\lim_{x\to 2^-}-(x+1)$$

$$=-3$$

Absolute value does nothing for $x>2$

$$\lim_{x\to 2^+}\frac{|x-2|(x+1)}{x-2}=\lim_{x\to 2^+}\frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}}=\lim_{x\to 2^+}(x+1)=3$$

So since $\lim_{x\to 2^-} f(x)\neq\lim_{x\to 2^+} f(x)$, we know $\lim_{x\to 2} f(x)$ does not exist, and thus isn't equal to $f(2)$, so it isn't continuous there.

b) Is there any value of a for which the function $g(x)=\dfrac{|x-a|(x+1)}{x-a}$ will be continuous at every real number?

Nope. Even if you get $\lim_{x\to a^-} g(x)=\lim_{x\to a^+} g(x)$, you'll still have $g(a)$ undefined, so you won't have $\lim_{x\to a} g(x)=g(a)$. □