Exam 4 Calc 1 12/2/2011

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Approximate $\int_1^2 \ln x\,dx$ using a left-hand sum with $n = 3$ subdivisions.

$\frac{2-1}{3}$ $\quad \frac{1}{3} = \Delta x$

$x_0 = 1 \qquad \ln(1) = 0$ Great

$x_1 = 1+\frac{1}{3} \qquad \ln(1\frac{1}{3})\ \ln(\frac{4}{3}) \approx .2876820725$

$x_2\ \ 1+\frac{2}{3} \qquad \ln(1\frac{2}{3}) \approx .5108256238$

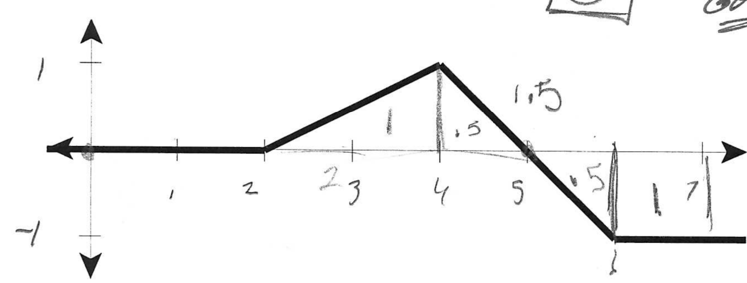
approximation!

$\left(\ln(1) + \ln(\frac{4}{3}) + \ln(\frac{5}{3})\right)\cdot \boxed{\Delta x \approx .266169235}$

2. the function $f(x)$ whose graph is shown below, find

a) $\int_0^4 f(x)\,dx$ $\quad (\text{base } 2)\cdot\frac{1}{2}\cdot h1 = 1\cdot 1 = \boxed{1}$

b) $\int_0^7 f(x)\,dx$ $\quad 1.5 - 1.5 = \boxed{0}$ Good!



3. Evaluate $\int \frac{x}{\sqrt[3]{x+4}}\,dx$.

$\int \frac{x}{\sqrt[3]{x+4}}\,dx = \int \frac{x}{u^{1/3}}\,du$ $\quad u = x+4 \quad x = u-4$

$du = dx$

$= \int \frac{u-4}{u^{1/3}}\,du = \int \frac{u}{u^{1/3}} - \frac{4}{u^{1/3}}\,du = \int u^{2/3} - 4u^{-1/3}\,du$

$= \frac{3}{5}u^{5/3} - 6u^{2/3}$

$= \boxed{\frac{3}{5}(x+4)^{5/3} - 6(x+4)^{2/3} + C}$

Well done

4. Suppose $\int_0^3 f(x)\,dx = 2$, $\int_3^6 f(x)\,dx = -5$, and $\int_3^6 g(x)\,dx = 1$.

a) Evaluate $\int_0^3 5f(x)\,dx$. $= 5\int_0^3 f(x)\,dx = 5(2) = 10$

b) Evaluate $\int_3^6 (3f(x) - g(x))\,dx$. $= \int_3^6 (3f(x))\,dx - \int_3^6 g(x)\,dx$

$= 3\int_3^6 f(x)\,dx - \int_3^6 g(x)\,dx = 3(-5) - 1 = -16$

Good!

c) Evaluate $\int_6^3 (f(x) + 2g(x))\,dx$. $= -\int_3^6 (f(x) + 2g(x))\,dx$

$= -\int_3^6 f(x)\,dx - 2\int_3^6 g(x)\,dx$

$= -(-5) - 2(1) = 5 - 2 = 3$

5. a) Evaluate $\frac{d}{dx}\int_0^x \cos(t^2)\,dt$. $= \cos x^2$

By F.T.C. 1

b) Evaluate $\frac{d}{dx}\int_0^{x^2} \cos(t^2)\,dt$. $= \cos(x^2)^2 \cdot 2x$

By F.T.C. 2

6. Find the average value of $f(x) = \sin x$ on the interval $[0,\pi]$.

$$\frac{1}{\pi - 0}\int_0^\pi \sin x\,dx = \frac{1}{\pi}\cdot\Big[-\cos x\Big]_0^\pi$$

$$\frac{1}{\pi}\big[1 - -1\big] = \frac{2}{\pi}$$

Nice

7. Evaluate $\displaystyle\int_1^4 \frac{1+\sqrt{x}}{x}\,dx$.

$\int \frac{1}{x} + \frac{\sqrt{x}}{x}$

$\int \frac{1}{x} = \ln x$ $\quad \int \frac{\sqrt{x}}{x} = \int \frac{1}{\sqrt{x}} = 2\sqrt{x}$

$\int \frac{1+\sqrt{x}}{x} = \ln x + 2\sqrt{x}$

$\int_1^4 \frac{1+\sqrt{x}}{x} = \left[\ln x + 2\sqrt{x}\right]_1^4 = (\ln 4 + 2\sqrt{4}) - (\ln 1 + 2\sqrt{1})$

$(\ln 4 + 4) - (0 + 2\sqrt{1})$

$\boxed{\ln 4 + 4 - 2\sqrt{1}}$

Good

8. Bunny is a Calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is so confusing. I get that, like, these integral things are supposed to give you an area, right? But so there were these true/false questions on our exam, because they can grade them by computer for, like, a thousand students, right? So one of them was to say whether if the integral of some function on this interval was zero, did the function have to be zero too everywhere on that interval. I figured yes, because if the function has a height of zero then the area under it is zero too, right? But they said false."

Help Bunny by explaining how a definite integral can be zero without the function having a height of zero everywhere on the interval.

On the graph of a function, the area above the x-axis is counted as positive and the area below the x-axis is counted as negative. So on an interval where the graph's negative and positive areas are equal, its total area (and its integral) will be zero. Take the sine function:

on the interval $[0, 2\pi]$, the areas above and below the graph are equal, and the height is not zero at all points like Bunny thought. Let's see if the integral is zero.

$$\int_0^{2\pi} \sin x \, dx = \Big[-\cos x\Big]_0^{2\pi}$$ integrate

$= -\cos 2\pi + \cos 0$ Fundamental Theorum of Calculus

$= -1 + 1$ evaluate cosines

$= 0$ simplify.

Excellent!

This is just one of many functions with an integral of zero on some interval where the height is not always zero because the positive and negative areas are equal.

9. Write in sigma notation a right-hand Riemann approximation with n subdivisions to the integral $\int_1^3 \frac{1}{x^3+1}\,dx$.

$\Delta x = \frac{2}{n}$

$x_i = \left(1 + \frac{2i}{n}\right)$

Excellent!

$$\boxed{\sum_{i=1}^{n} \frac{1}{\left(1+\frac{2i}{n}\right)^3+1} \cdot \left(\frac{2}{n}\right)}$$

10. In Calculus 2 it turns out that the integral $\int_0^R 4\pi x\sqrt{R^2 - x^2}\,dx$ is of interest. Evaluate this integral.

Let $u = R^2 - x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2x} = dx$

$$= \int_{R^2}^{0} 4\pi x \cdot u^{1/2} \cdot \frac{du}{-2x}$$

$$= -2\pi \int_{R^2}^{0} u^{1/2}\,du$$

$$= -2\pi \cdot \frac{2}{3} u^{3/2} \Big|_{R^2}^{0}$$

$$= -\frac{4\pi}{3} \cdot \left(0 - (R^2)^{3/2}\right)$$

$$= \frac{4}{3}\pi R^3$$