

Exam 2 Calc 3 10/28/2011

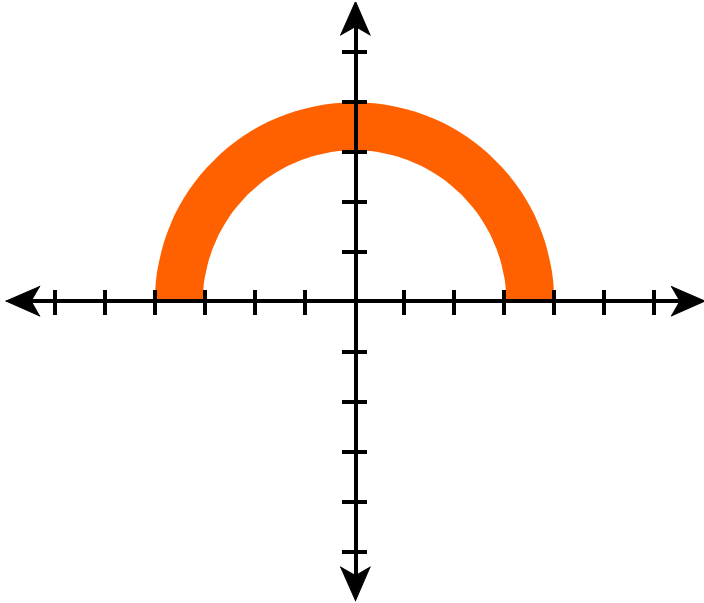
Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Set up an iterated integral for the volume under $f(x, y) = 3x^2 + 5x - 2y + 10$ and above the rectangle with vertices $(2,0)$, $(2,3)$, $(0,3)$, and $(0,0)$.

2. The table below shows data from a population survey done on brown barbaloots in a region $R = [0,8] \times [0,4]$, given in barbaloots per square furlong. Estimate the total brown barbaloot population in this region using the Midpoint Rule with $m = n = 2$.

$x \backslash y$	0	1	2	3	4
0	12	14	15	14	7
2	13	16	21	20	11
4	14	17	23	19	12
6	11	13	21	18	9
8	8	9	10	7	5

3. Set up an iterated integral for the volume below $z = x^2y$, above the region shown below. Set up in terms of a single coordinate system, i.e., if you use cylindrical your integral should involve no x or y , etc.



4. Set up an iterated integral for the volume of the region under the plane $x + 2y - z = 0$ and above the region bounded by $y = x$ and $y = x^4$.

5. Show that the Jacobian for converting from rectangular to polar coordinates is r .

6. Set up an iterated integral for $\iiint_E xyz \, dV$, where E lies between spheres with radius 2 and 4, both centered at the origin, and above the cone $z = \sqrt{x^2 + y^2}$. Set up in terms of a single coordinate system, i.e., if you use cylindrical your integral should involve no x or y , etc.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this Calc 3 stuff is soooo confusing! Like, we just started doing these double integral thingies, right? And the professor was, like, making fun of this girl for asking why if you go from -2 to 2 and -2 to 2 , you don't get the volume of a paraboloid that's, like, $z = 4 - x^2 - y^2$. I mean, if you do less than that, you're leaving out part of the paraboloid, right?"

Explain clearly to Bunny what goes wrong when trying to find the volume of her paraboloid that way.

8. Set up a triple integral for the volume of the portion of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ which lies above the plane } z = c/2.$$

9. Suppose that space aliens infuse the atmosphere with sillydust in such a way that the density of sillydust varies linearly with latitude from α ounces per cubic mile of atmosphere at the north pole to β ounces per cubic mile at the south pole, and drops off steadily to 0 at an altitude of 1 mile above the planet's surface. Modeling the Earth as a sphere with radius 4000 miles, set up an iterated integral for the amount of sillydust in Earth's atmosphere.

10. Set up integrals for the x coordinate of the centroid of the wedge of the cylinder $x^2 + y^2 = 1$ sliced off by the planes $z = 1 - x$ and $z = x - 1$.

Extra Credit (5 points possible): Set up integrals for the x coordinate of the centroid of the wedge of the cylinder $x^2 + y^2 = 4$ below the plane $z = 1 - x$ and above the plane $z = x - 1$.

Your score on question 11 can replace your lowest score among questions 8-10, if desired.

11. Evaluate the integral $\iint_R (x - 3y) dA$, where R is the triangular region with vertices $(0,0)$, $(2,1)$, and $(1,2)$, using the transformation $x = 2u + v$, $y = u + 2v$.