**Exam 3 Calc 3 12/2/2011**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F}(x,y) = \langle 6xy, 3x^2 + 3y^2 \rangle$ and C is the first-quadrant portion of a circle (centered at the origin) from (3,0) to (0,3).

2. Show that there is no vector field $\mathbf{G}$ such that curl $\mathbf{G} = 2x\,\mathbf{i} + 3yz\,\mathbf{j} - xz^2\,\mathbf{k}$.

3. Compute $\int_C \langle 4x+1, x-y \rangle \cdot d\vec{r}$ for a line segment beginning at (3, 0) and ending at (1, 2).

4. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = 3xy\mathbf{i} + 6x^2\mathbf{j}$ and C is the path consisting of four line segments joining the points (0,0), (3,0), (3,2), and (0,2) in that order.

5. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = \langle 4x, 7y, -3z \rangle$ and $C$ is the boundary of the first-octant portion of a sphere with radius 5 (centered at the origin).

6. Show that for any vector field in $\mathbb{R}^3$ whose component functions have continuous second-order partial derivatives, div curl $\mathbf{F} = 0$. Make it clear why the requirement about continuity is important.

7. Suppose that $S_1$ is the portion of the cylinder $x^2 + y^2 = 1$ between $z = 0$ and $z = 3$, and that $S_2$ is the portion of the cylinder $x^2 + y^2 = 1$ between $z = 3$ and $z = 6$, in both cases with outward orientation.

   a) **Describe** the boundaries of $S_1$ and $S_2$ clearly, and **explain** which orientations for those boundaries count as positive.

b) If $S$ is the portion of the cylinder $x^2 + y^2 = 1$ between $z = 0$ and $z = 6$, with outward orientation, **explain** what connection there is between $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$, $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$, and $\iint_{S} \mathbf{F} \cdot d\mathbf{S}$.

8. Compute $\iint_{S_L} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = 2x\mathbf{i} + 2y\mathbf{j} + \mathbf{k}$ and $S_L$ is the portion of $z = x^2 + y^2$ between the planes $z = 1$ and $z = 9$, with upward orientation.

9. Compute $\iint_{S_a} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = 2x\mathbf{i} + 2y\mathbf{j} + \mathbf{k}$ and $S_a$ is a disc of radius $a$ centered on the point $(0,0,a^2)$ in the plane $z = a^2$, with upward orientation.

10. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = 2x\mathbf{i} + 2y\mathbf{j} + \mathbf{k}$ and S is the portion of $z = x^2 + y^2$ between the planes $z = 1$ and $z = 9$ along with a disc of radius 1 centered on the point (0,0,1) in the plane $z = 1$ and a disc of radius 3 centered on the point (0,0,9) in the plane $z = 9$, all with outward orientation.

Extra Credit (5 points possible):
Show that if $f(x, y,z)$ is a scalar function and $\mathbf{G}(x,y,z)$ is a vector field, then

$$\text{div}(f\,\mathbf{G}) = f\,\text{div}\,\mathbf{G} + \mathbf{G} \cdot \nabla f$$