Quiz 6 **Calculus 3** **11/18/2011**

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Compute the curl of the vector field $\mathbf{F}(x,y,z) = \langle x^2 y, x^2 z, x^3 \rangle$.

$$\text{curl}(\vec{F}) = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \langle x^2 y, x^2 z, x^3 \rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & x^2 z & x^3 \end{vmatrix} = \left(\frac{\partial x^3}{\partial y} - \frac{\partial (x^2 z)}{\partial z}\right)\vec{i} - \left(\frac{\partial x^3}{\partial x} - \frac{\partial x^2 y}{\partial z}\right)\vec{j} + \left(\frac{\partial (x^2 z)}{\partial x} - \frac{\partial (x^2 y)}{\partial y}\right)\vec{k}$$

$$= (0 - x^2)\vec{i} - (3x^2 - 0)\vec{j} + (2xz - x^2)\vec{k}$$

$$= \langle -x^2, -3x^2, 2xz - x^2 \rangle$$

Great!

2. Compute the divergence of the vector field $\mathbf{F}(x,y,z) = \langle x^2 y, x^2 z, x^3 \rangle$. $\text{div}\,\vec{F} = \nabla \cdot F$

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle x^2 y, x^2 z, x^3 \rangle$$

$$= 2yx + 0 + 0$$

$$\text{div}\,\vec{F} = 2yx$$

Wow!