

Quiz 6 Calculus 3 11/18/2011

Each problem is worth 5 points. Clear and complete justification is required for full credit.

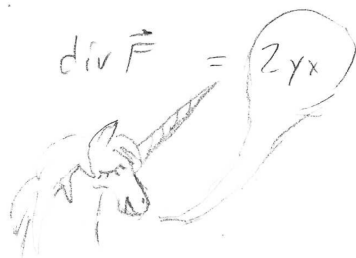
1. Compute the curl of the vector field $\mathbf{F}(x,y,z) = \langle x^2y, x^2z, x^3 \rangle$.

$$\begin{aligned} \text{curl}(\vec{F}) &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \langle x^2y, x^2z, x^3 \rangle \\ &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & x^2z & x^3 \end{vmatrix} = \left(\frac{\partial x^3}{\partial y} - \frac{\partial(x^2z)}{\partial z} \right) \vec{i} - \left(\frac{\partial x^3}{\partial x} - \frac{\partial(x^2y)}{\partial z} \right) \vec{j} \\ &\quad + \left(\frac{\partial(x^2z)}{\partial x} - \frac{\partial(x^2y)}{\partial y} \right) \vec{k} \\ &= (0 - x^2) \vec{i} - (3x^2 - 0) \vec{j} + (2xz - x^2) \vec{k} \\ &= \langle -x^2, -3x^2, 2xz - x^2 \rangle \quad \text{Great!} \end{aligned}$$

2. Compute the divergence of the vector field $\mathbf{F}(x,y,z) = \langle x^2y, x^2z, x^3 \rangle$. $\text{div} \vec{F} = \nabla \cdot \mathbf{F}$

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle x^2y, x^2z, x^3 \rangle$$

$$= 2yx + 0 + 0$$

$$\text{div} \vec{F} = 2yx$$


Wow!