Exam 2 Calc 1 10/12/2012

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the derivative of a function $f(x)$.

2. If $f(x) = x^3 - 5x$, write an equation for the line tangent to f at the point $(2,-2)$.

3. Space aliens are going to destroy planet Earth unless you can demonstrate there's intelligent life here by showing that $\left(\tan x\right)' = \sec^2 x$. [Hint: Remember tan x = sin x / cos x.]

4. Let $h(x) = f(g(x))$ and $q(x) = f(x)/g(x)$. Use the table below to compute

   a) $h'(2)$

   b) $q'(3)$

x	1	2	3	4	5
$f(x)$	0	3	5	1	0
$f'(x)$	5	2	–5	–8	–10
$g(x)$	1	5	7	3	2
$g'(x)$	2	10	20	15	20

5. [Adapted from B&C §3.5] Suppose that a stone is thrown vertically upward from the edge of a cliff with an initial velocity of 96 ft/s from a height of 64 ft above the ground. The height s (in feet) of the stone above the ground t seconds after it is thrown is $s = -16t^2 + 96t + 64$.

   a) Determine the velocity v of the stone after t seconds

   b) When does the stone reach its highest point?

   c) What is the height of the stone at the highest point?

6. State and prove the Quotient Rule for derivatives. Make it clear how you use any assumptions.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod. This Calculus stuff is *soooooo* confusing. I mean, I get it when they say the derivative of sin x is cos x and stuff like that, right? But then instead of just a list of stuff like that we need to know, our professor said we should be able to figure out derivatives of inverses of any functions we know derivatives for. How could we do that?"

   Help Bunny by explaining as clearly as you can how knowing the derivative of a function can allow you to find the derivative of that function's inverse function.

8. Use the definition of the derivative to show that the derivative of $f(x) = \sqrt{x}$ is $f'(x) = \frac{1}{2\sqrt{x}}$.

9. Find an equation of the line tangent to the curve $x^2y + y^3 = 39$ at the point (2,3).

10. Let $s(x) = \frac{e^x + e^{-x}}{2}$ and $c(x) = \frac{e^x - e^{-x}}{2}$. Find the derivatives of s and of c and describe what's going on.

Extra Credit (5 points possible):
For the function $s(x)$ from #10, what can you say about the derivative of $s^{-1}(x)$?