

Exam 4 Calc 1 12/7/2012

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Evaluate $\int_1^3 x^2 dx$.

2. Find the average value of $f(x) = \sin x$ on the interval $[0, \pi]$.

3. If you use a left-hand sum with $n = 4$ subdivisions to approximate $\int_1^5 \frac{1}{x} dx$, what are:

$$\Delta x =$$

$$\bar{x}_1 =$$

$$\bar{x}_2 =$$

$$\bar{x}_3 =$$

$$\bar{x}_4 =$$

$$f(\bar{x}_1) =$$

$$f(\bar{x}_2) =$$

$$f(\bar{x}_3) =$$

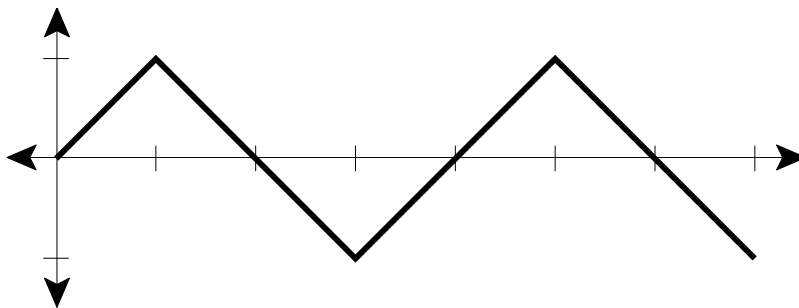
$$f(\bar{x}_4) =$$

$$\sum_{k=1}^4 f(\bar{x}_k) \cdot \Delta x =$$

4. For the function $f(x)$ whose graph is shown below, find

a) $\int_0^2 f(x) dx$

b) $\int_0^6 f(x) dx$



5. Evaluate $\int \frac{x}{\sqrt{9-x^2}} dx$.

6. Suppose $\int_0^2 f(x) dx = 5$, $\int_2^5 f(x) dx = -1$, $\int_0^2 g(x) dx = 7$, and $\int_2^5 g(x) dx = 2$.

a) Evaluate $\int_0^2 3f(x) dx$.

b) Evaluate $\int_0^5 f(x) dx$.

c) Evaluate $\int_2^5 [f(x) - g(x)] dx$.

7. a) Evaluate $\frac{d}{dx} \int_0^x \frac{1}{1+t^3} dt$.

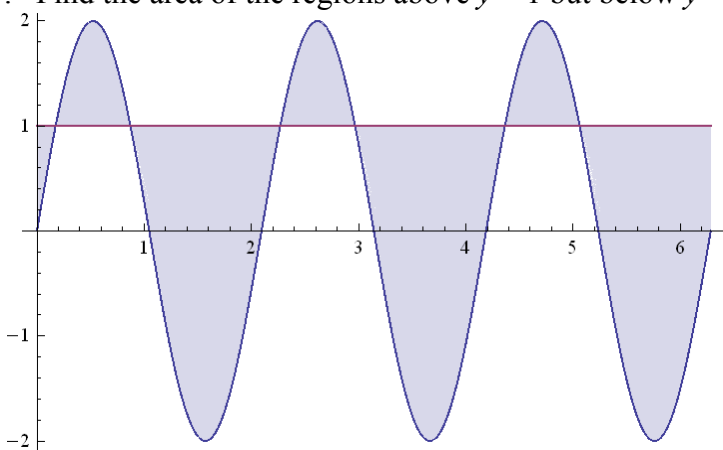
b) Evaluate $\frac{d}{dx} \int_0^{5x} \frac{1}{1+t^3} dt$.

8. The first edition of the Cliff'sNotes Calculus Quick Review says on page 116 that the area A of the region bounded by the graph of $f(x)$, the x axis, and the lines $x = a$ and $x = b$ is given by

$$\left| \int_a^b f(x) dx \right| \text{ when the function } f(x) \text{ is sometimes above and sometimes below the } x\text{-axis.}$$

Explain in at least a few sentences why the absolute value bars in this expression do or don't accurately express the actual area bounded by these curves.

9. Find the area of the regions above $y = 1$ but below $y = 2\sin(3x)$ between $x = 0$ and $x = 2\pi$.



10. Evaluate $\int_0^5 (x+1)\sqrt{25-x^2} dx$.

Extra Credit (5 points possible):

Find the value of c such that the areas of the two regions shown between $y = c$ and $y = x^2 - 3x^3$ (but to the right of $x = 0$) are equal.

