Exam 3   Calc 3   11/30/2012

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Let $\mathbf{F}(x,y) = \langle 5x^4y^2, 2x^5 y\rangle$, and C be the line segment from (2,–1) to (0,2).
   Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$.

2. Let $\mathbf{F}(x, y, z) = \langle 5, 2xy, x - y + z\rangle$. Find div $\mathbf{F}$.

3. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = 3xy\mathbf{i} + 6x^2\mathbf{j}$ and C is the path consisting of four line segments joining the points (0,0), (3,0), (3,2), and (0,2) in that order.

4. Let $\mathbf{F}(x,y) = \langle xy + x^2, y^2 \rangle$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for C the second-quadrant portion of a circle with radius 3 centered at the origin, traversed counterclockwise.

5. Let $\mathbf{F}(x, y, z) = \langle 0,0,-1 \rangle$, and S be the portion of $z = 4 - x - y$ in the first octant, oriented upward. Evaluate $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$.

6. Show that for any scalar function $f(x, y, z)$ with continuous second-order partial derivatives, $\text{curl}(\nabla f) = \mathbf{0}$. Make it clear why the requirement about continuity is important.

7. Biff is a student at Enormous State University, and he's having some trouble. Biff says "Man, I was doing okay with these line integrals when they just told us which way to do 'em, but then for the exam it turns out they don't tell you which section a problem is from, so you actually have to think for yourself. I hate that. So what I've got to figure out is how to know when I can use that fundamental theory thing for line integrals instead of doing them the long way. Got any advice?"

   Help Biff out by explaining as clearly as possible when he can use the Fundamental Theorem for Line Integrals on a problem.

Extra Credit (5 points possible):

Show that if $f(x, y,z)$ is a scalar function and $\mathbf{G}(x,y,z)$ is a vector field, then

$$\operatorname{div}(f\,\mathbf{G}) = f \operatorname{div} \mathbf{G} + \mathbf{G} \cdot \nabla f$$