Exam 1A   Real Analysis 1   10/5/2012

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of the limit of a sequence a_n.

2. State the definition of a Cauchy sequence.

3. State the definition of a function diverging to $-\infty$ as x approaches a from the right.

4. Give an example of a sequence that diverges to $+\infty$ but is not eventually increasing.

5. a) State the Bolzano-Weierstrass Theorem for Sequences

   b) State the Cauchy Convergence Criterion.

6. Suppose that f and g are functions with both having domain $D \subseteq \mathbb{R}$. Prove that if $\lim_{x \to +\infty} f(x) = A$ and $\lim_{x \to +\infty} g(x) = B$ then $\lim_{x \to +\infty} (f \cdot g)(x) = A \cdot B$.

7. State and prove the Monotone Convergence Theorem.

8. Why does the definition of a limit as x approaches a need to require that δ be greater than zero?

9. Suppose that a_n is a sequence with domain $\mathbb{N}$. Is the condition that $\forall n \in \mathbb{N}, a_n > n$ equivalent to saying a_n is unbounded?

10. Suppose that a_n is a sequence whose domain is $\mathbb{N}$, and $S = \{a_n \mid n \in \mathbb{N}\}$ has an accumulation point α. Does there necessarily exist a sequence b_n of values from S which converges to α? Why or why not?