Problem Set 6 Real Analysis Due 10/17/2012

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. We say that a sequence is CCauchy iff $\forall \varepsilon > 0\ \exists n, m$ such that $|a_m - a_n| < \varepsilon$. Give an example of a sequence which is CCauchy but not Cauchy.

2. We say that a sequence is CCCauchy iff $\forall \varepsilon > 0\ \forall n, m \in \mathbb{N}$ we have $|a_m - a_n| < \varepsilon$. Give an example of a sequence which is Cauchy but not CCCauchy.

3. Show that if $f(x)$ is continuous at a, and $c \in \mathbb{R}$ is a constant, then $c \cdot f(x)$ is continuous at a.

4. Show that if $f(x)$ and $g(x)$ are continuous at a, then $(f + g)(x)$ is continuous at a.

5. Show that if $f(x)$ and $g(x)$ are continuous at a, then $(f \cdot g)(x)$ is continuous at a.

6. Do Exercise 6(k) in §4.1.