Exam 1a   Calc 3   9/27/2013

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function f(x, y) with respect to y.

2. Find an equation for the plane tangent to $z = 4 - 2x^2 - y^2$ at the point $(-1,1,1)$.

3. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$ does not exist.

4. Suppose that f is a function of w, x, y, and z, each of which is a function of t. Write the Chain Rule formula for $\frac{df}{dt}$. Make very clear which derivatives are partials.

5. Let $f(x,y)=\sqrt{y^2-x-1}$. Find the directional derivative of f at the point (–4,1) in the direction of the vector $\langle 1,2 \rangle$.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this Calc 3 stuff is killing me. There's all this stuff where you can't just solve an equation and have an answer, you know? It's totally unfair. Like, there was this question they asked us when we reviewed in class, about like, if there was this crit point where the second derivative test is 0 so it's inconclusive, but if you knew the function was higher for every point on a circle with radius 1 centered at the critical point, would that guarantee it was a local min. There's not even a freaking formula, so how should I know?"

   Explain clearly to Biff whether the conditions he describes are sufficient to draw a conclusion, and why.

8. Find the maximum value of $f(x, y) = xy - y + 3$ subject to the constraint $3x + 2y = 6$.

9. Find and classify all critical points of the function $f(x, y) = x^2y^2 - x^2 - y^2 + 1$.

10. Find all points on the surface $f(x,y) = x^2y + y^3/3$ where the directional derivative is greatest in the direction $\langle 1,1 \rangle$.

Extra Credit (5 points possible):
Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin. [Stewart 7th, p. 954]