## Exam 2a Calc 3 10/25/2013

Each problem is worth 10 points. For full credit provide complete justification for your answers. All integrals should be set up in terms of a single coordinate system, i.e., if you use cylindrical your integral should involve no $x$ or $y$, etc.

1. Approximate the volume under $z=\sqrt{16-x^{1.5}-y^{1.5}}$ and above the rectangle with vertices $(0,0),(4,0),(4,2)$, and $(0,2)$ using a midpoint double Riemann sum with $n=m=2$ subdivisions.
2. Set up an iterated integral for the volume below $z=f(x, y)$ on the region $R$ pictured below:

3. Set up an iterated integral for the volume below $z=x^{2} y$, above the region shown below.

4. Set up an iterated integral for the volume of the region beneath the surface $z=x y+10$ and above the annular region outside $x^{2}+y^{2}=4$ but inside $x^{2}+y^{2}=16$.
5. Evaluate $\int_{0}^{1} \int_{y}^{1} e^{x^{2}} d x d y$.
6. Many calculus texts include the formula $\bar{y}=\frac{1}{A} \int_{a}^{b} \frac{1}{2}\left\{[f(x)]^{2}-[g(x)]^{2}\right\} d x$ for the $y$ coordinate of the centroid of a region which lies between two curves $y=f(x)$ and $y=g(x)$, where $f(x)>g(x)$, and between $x=a$ and $x=b$. Use iterated integrals to justify this formula.
7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this Calc 3 stuff is soooo confusing! Like, our exam review sheet has a bunch of true/false questions, right? And one of them was, like, if you know a function never gets above 1 , then do you know its integral is less than 1 too? I've got no clue how you could tell if they don't give you a formula for the function."

Explain clearly to Bunny what the correct answer to her question is, and why.
8. Find the Jacobian for converting from rectangular to spherical coordinates.
9. Set up iterated integrals for the $z$ coordinate of the center of mass of the cone below $z=25-\sqrt{x^{2}+y^{2}}$ but above $z=0$.
10. Set up an iterated integral (or integrals) for the volume of the region under both $z=1 / x$ and $z$ $=1 / y$, and between $z=0$ and $z=5$, and within 3 units of the $z$-axis.

Extra Credit (5 points possible):
Let $\llbracket x \rrbracket$ represent the greatest integer function, so it outputs the greatest integer that is less than or equal to $x$. What is $\iint_{R} \llbracket \sqrt{x^{2}+y^{2}} \rrbracket d A$, where $R$ is a circle with radius $a$ centered at the origin?

