

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{x+2y}{x-2y}$.

Approach along $x=0$

$$\lim_{(0,y) \rightarrow (0,0)} \frac{(0)+2y}{(0)-2y} = \lim_{y \rightarrow 0} \frac{2y}{-2y} = \lim_{y \rightarrow 0} -1 = \boxed{-1}$$

Approach along $y=0$

$$\lim_{(x,0) \rightarrow (0,0)} \frac{x+2(0)}{x-2(0)} = \lim_{x \rightarrow 0} \frac{x}{x} = \lim_{x \rightarrow 0} 1 = \boxed{1}$$

Approach along $y=x$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x+2(x)}{x-2(x)} = \lim_{x \rightarrow 0} \frac{3x}{-x} = \lim_{x \rightarrow 0} -3 = \boxed{-3}$$

∴ Since the limits for different approaches do not agree, the limit does not exist.

Excellent!