Exam 1 Calc 3 9/26/2014

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function $f(x, y)$ with respect to x.

2. Find an equation for the plane tangent to $z = x^2 + y^2$ at the point $(3,–2,13)$.

3. Show that $\lim_{(x,y)\to(0,0)} \dfrac{x-2y}{x+2y}$ does not exist.

4. Suppose that w is a function of x, y, and z, each of which is a function of s and t. Write the Chain Rule formula for $\dfrac{\partial w}{\partial s}$. Make very clear which derivatives are partials.

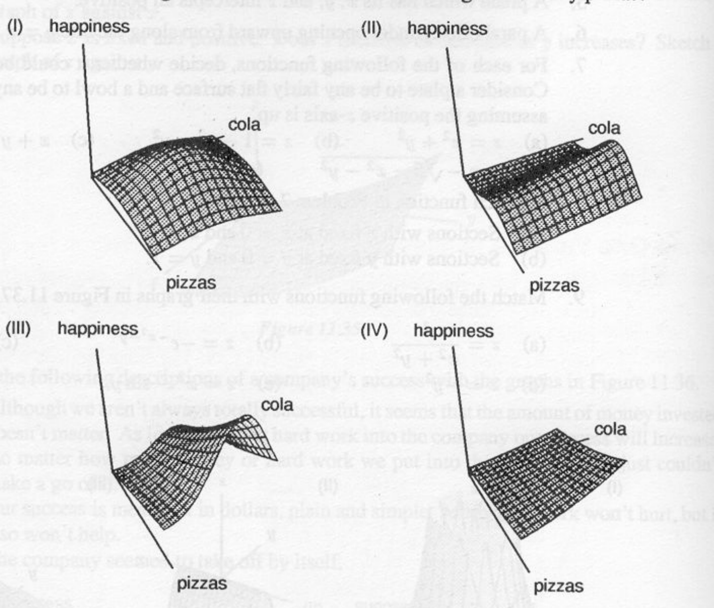
5. Let $f(x, y) = xy^2 - x^3$. At the point (1,2), in which direction is the directional derivative greatest?

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, this Calc 3 stuff is killing me. There was this problem on our first exam that made totally no sense. See, there's these graphs about pizza and cola, but there's no formulas at all. How the heck can you do math with just pictures?"

   Explain clearly to Biff **which** graph matches with each description, and **why**.

10. You like pizza and you like cola. Which of the graphs in Figure 11.38 represents yo
happiness as a function of how many pizzas and how much cola you have if
    (a) There is no such thing as too many pizzas and too much cola?
    (b) There is such a thing as too many pizzas or too much cola?
    (c) There is such a thing as too much cola but no such thing as too many pizzas?



8. Find the maximum value of $f(x, y) = x^2 + y$ subject to the constraint $x^2 + y^2 = 9$.

9. A road is to be built from Monkeysaddle Pass up to the new Herron's Roost Ski Resort. The surface in this vicinity happens to exactly match the function $f(x, y) = 0.1(xy^2 - x^3)$. Engineers are trying to determine whether the road passing through the point (1,2) can go in the direction $\langle 3,4 \rangle$.

   a) What is the slope of f at this point in this direction?

   b) In which direction(s), from this point, will the slope be exactly 0.1 (which, as a 10% grade, represents the steepest road many vehicles can manage)?

10. Find the maximum and minimum values of $f(x, y) = \dfrac{x}{1 + x^2 + y^2}$.

Extra Credit (5 points possible):

Find an equation for the line through the origin and the point (a, b, c). Determine where this line intersects the paraboloid $z = x^2 + y^2$, and describe the angle at which the line intersects the surface.