Exam 3a Calculus 3 11/25/2014

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

1. Parametrize and give bounds for a path C which traverses the left half of a circle (centered at the origin) counterclockwise from (0, 3) to (0, –3).

2. Let $\mathbf{F}$ be the vector field $\mathbf{F} = 2xy\,\mathbf{i} + x^2\,\mathbf{j}$. Let C be the line segment from (2,8) to the origin. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = 3y\,\mathbf{i} + 6x\,\mathbf{j}$ and C is the path consisting of four line segments joining the points (0,0), (5,0), (5,2), and (0,2) in that order.

4. Let **F** be the vector field $\mathbf{F} = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$. Let S be the sphere with radius 1, centered at the origin. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

5. Let **F** be the vector field $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + 2\,\mathbf{k}$. Let S be the disk $x^2 + y^2 \leq 9$ in the plane $z = 6$, with upward orientation. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

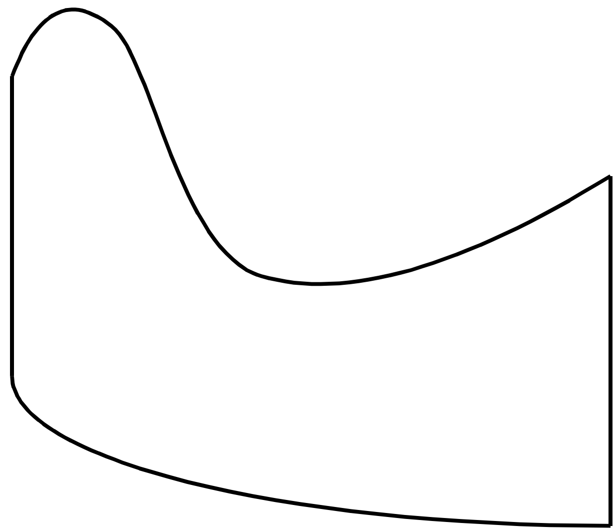
6. Prove that if $f(x,y,z)$ is a function with continuous second-order partial derivatives, then $\operatorname{curl}(\nabla f) = \mathbf{0}$. Make it clear how the requirement that the partials be continuous is important.

7. Bunny is a calc 3 student at a large state university and she's having some trouble. Bunny says "Ohmygod, I am so totally confused by this class. I mean, I can work out a lot of the problems, but I totally don't understand what any of it means. I guess it mostly doesn't really matter, since our exams are all multiple choice, but I really wish I understood something instead of just getting answers. Like, the professor was saying over and over yesterday that it should be clear why if a vector field is conservative then line integral-thingys on closed paths always come out zero, but he wouldn't say why – just that it was supposed to be clear! Why would that be clear?"

   Explain as clearly as possible to Bunny why line integrals on closed paths in conservative vector fields are always zero.

8. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \langle -x, y \rangle$ and C is a circular path (centered at the origin) beginning at (1,0) and traversing n quarter-circles (where, for instance, traversing 8 quarter-circles means passing twice around a circle).

9. Evaluate $\int_{C_1} P\,dx$ and $\int_{C_2} P\,dx$ for the region sketched below, given that C_2 is a curve for which $y = f_2(x)$ and C_1 is a vertical line.



10. Let $\mathbf{F}(x, y,z) = \langle 3,xy,z \rangle$. Evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where S is the top half of a sphere with radius 5 centered at the origin, using outward orientation.

Extra Credit (5 points possible):

If $\mathbf{a}$ is a constant vector, $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, and S is an oriented, smooth surface with simple, closed, smooth, positively oriented boundary curve C, show that

$$\iint_S 2\mathbf{a} \cdot d\mathbf{S} = \int_C (\mathbf{a} \times \mathbf{r}) \cdot d\mathbf{r}\,.$$