## Problem Set 4Real Analysis 1Due 9/24/14

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

- 1. Every Cauchy sequence is bounded. [No fair using the CCC!]
- 2. If  $\{a_n\}$  is a Cauchy sequence and  $S = \{a_n | n \in \mathbb{N}\}$  is finite, then  $\{a_n\}$  is constant from some point on.
- 3. Let  $s_0$  be an accumulation point of *S*. Then any neighborhood of  $s_0$  contains at least one point of *S* different from  $s_0$  iff any neighborhood of  $s_0$  contains infinitely many points of *S*.
- 4. The sum of two Cauchy sequences is Cauchy. [No fair using the CCC!]
- 5. A finite set has no accumulation points.
- 6. Give an example of a sequence where  $\lim_{n\to\infty} |a_{n+1}-a_n| = 0$ , but the sequence diverges.
- 7. Let  $a_{n+1} = 1 + \frac{a_n}{2}$ , and  $a_1 = 1$ . Then  $\{a_n\}$  converges.
- 8. Suppose  $\{a_n\}$  and  $\{b_n\}$  both oscillate. Then  $\{a_n \cdot b_n\}$  and  $\{a_n + b_n\}$  oscillate.