Exam 3 Calc 1 11/13/2015

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find all critical points of $f(x) = x^3 - 12x$.

2. Determine where $f(x) = x^3 - 12x$ has local maxima and minima.

3. Find the linear approximation L(x) for $f(x) = \sqrt[4]{x}$ at 16.

4. Find two positive real numbers such that they add to 40 and their product is as large as possible.

5. Evaluate $\lim_{x\to\infty} \frac{\ln x}{x}$.

6. Apply Newton's Method to $f(x) = x^3 - 5$ with initial guess $x_0 = 2$ to calculate x_1 .

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "I think calculus is the hardest thing ever! Just when you think it's getting easy, like with the Loopy-tall rule thingy? At first I thought it was really easy. But then there was this one on the test, like a limit for $x^2 - 2x$ over 3x - 2, like if x is 0, and I did the Loopy-tall thingy and they gave me no credit, not even partial. That's so unfair!"

Explain clearly to Bunny what she should understand about using L'Hôpital's Rule on her function..

8. The manager of a large apartment complex knows from experience that 90 units will be occupied if the rent is 344 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 4 dollar increase in rent. Similarly, one additional unit will be occupied for each 4 dollar decrease in rent. What rent should the manager charge to maximize revenue?

9. Find all critical points of $f(x) = \sin^2 x$ and classify them as maxima, minima, or neither.

10. Find the points of inflection and the intervals on which $y = \frac{\ln x}{x}$ is concave up and concave down.

Extra Credit (5 points possible): Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{x} \right)^x$.