Exam 1 Calc 3 10/2/2015

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function $f(x, y)$ with respect to x.

2. Suppose that w is a function of x, y, and z, each of which is a function of s and t. Write the Chain Rule formula for $\frac{\partial w}{\partial s}$. Make very clear which derivatives are partials.

3. Find an equation for the plane tangent to $z = \dfrac{x - y}{x^2 + y^2}$ at the point (1, 2, –1/5).

4. Show that $\lim_{(x,y)\to(0,0)} \dfrac{x^6 + 9x^4y^2 - 9x^2y^4 - y^6}{\left(x^2+y^2\right)^3}$ does not exist.

5. Let $f(x, y) = 3x^2y - x^3$.
   a) At the point (1,2), find the directional derivative in the direction of the vector $\langle -3, 4 \rangle$.

   b) In which direction is the directional derivative greatest, and how large is the directional derivative in that direction?

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Omygod, this Calc 3 stuff is killing me. I understood pretty well about derivatives, because they're just slopes, and that's okay, right? But ohmygod, now there's an x derivative and a y one too? Ohmygod. I mean, just, ohmygod. But don't they have to be the same? I mean, like, if it's sloping up, then it's sloping up, right? It's not like the x slope could be positive if the y slope is negative or something, right?"

   Explain clearly to Bunny whether the derivatives with respect to x and y can actually have different signs, and why.

8. Find the minimum value of $f(x, y) = 2x^2 + 3y^2$ subject to the constraint $4x + 2y = 10$.

9. Find and classify all critical points of $f(x, y) = x^4 - 2x^2 - y^3 + 3y$.

10. Find an equation for the plane tangent to $f(x, y) = \sqrt{ax^2 + by^2}$ at the point (x_0, y_0). Show that regardless of choice of x_0 and y_0, such a plane will always pass through the origin.

Extra Credit (5 points possible):
A rectangle with length L and width W is cut into four smaller rectangles by two lines parallel to the sides. Find the maximum and minimum values for the sum of the squares of the areas of the smaller rectangles.