Exam 3 Calculus 3 12/4/2015

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

1. Parametrize and give bounds for a line segment C from (–2, 3) to (1, 2).

2. Let **F** be the vector field $\mathbf{F}(x, y) = (2xy - 1)\,\mathbf{i} + (x^2 + 3)\,\mathbf{j}$. Let C be the line segment from (5,2) to (5, 3), followed by the line segment from (5, 3) to (1, 3). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3. Evaluate $\int_C \left(x^2 - y\right) dx + x\, dy$, where C is the circle $x^2 + y^2 = 9$ with counterclockwise orientation.

4. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = \langle 4x, 7y, -3z \rangle$ and C is the boundary of the first-octant portion of a sphere with radius 5 (centered at the origin).

5. Let $\mathbf{F}(x,y) = \langle 4x - 1, y - x^2 \rangle$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for C the second-quadrant portion of a circle with radius 2 centered at the origin, traversed counterclockwise.

6. Prove that if **F**(x,y,z) is a vector field whose component functions have continuous second-order partial derivatives, then div(curl **F**) = 0. Make it clear how the requirement that the partials be continuous is important.

7. Bunny is a calc 3 student at a large state university and she's having some trouble. Bunny says "Ohmygod, I am so totally confused by this class. I mean, I can work out a lot of the problems, but I totally don't understand what any of it means. I guess it mostly doesn't really matter, since our exams are all multiple choice, but I really wish I understood something instead of just getting answers. They keep talking about path independence, right? But I totally can't believe they tried, like, every single path ever, right? So how can they know it won't matter if you do a different path than all the ones they tried?"

   Explain as clearly as possible to Bunny how it can be known that some line integrals are path independent.

8. Let $\mathbf{F}(x,y,z) = \langle 3, 2, -4\rangle$. Let S be the sphere $x^2 + y^2 + z^2 = 16$. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

9. Let $\mathbf{F}(x,y,z) = \langle 2z, -4x, 3y \rangle$. Let S be the cap of the sphere $x^2 + y^2 + z^2 = 16$ above the plane $z = \sqrt{7}$. Evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

10. A Calculus book[1] includes the formula

$$\int_S \vec{F} \cdot d\vec{A} = \int_R F(x, y, f(x, y)) \cdot \left(-f_x \vec{i} - f_y \vec{j} + \vec{k}\right) dx\, dy$$

in a box, along with some mumbo-jumbo about "Suppose the surface S is the part of the graph of $z = f(x, y)$ above a region R in the xy-plane, and suppose S is oriented upward." Why does this formula make sense?

[1]Hughes-Hallett, Gleason, McCallum, et al. 3rd, p.874.

Extra Credit (5 points possible): Evaluate $\int_0^a \int_0^b e^{\max\left\{b^2x^2, a^2y^2\right\}} dydx$, where a and b are positive.