Fake Quiz 1 Calc 3 10/23/2015

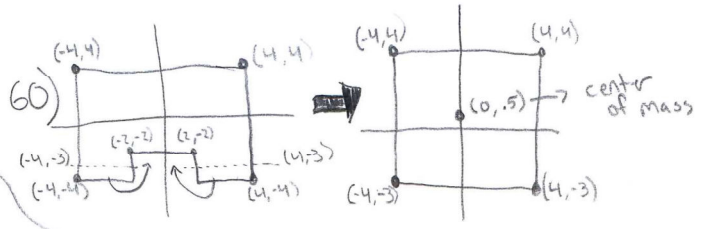
This is a fake quiz, this is *only* a fake quiz. In the event of an actual quiz, you'd have been given fair warning. Repeat: This is *only* a fake quiz.

1. Find the center of mass of the cardioid $r = 1 - \sin\theta$.

$$\bar{x} = \frac{\int_0^{2\pi}\int_0^{1-\sin\theta} x \cdot r\,dr\,d\theta}{\int_0^{2\pi}\int_0^{1-\sin\theta} r\,dr\,d\theta} = 0$$

$$\bar{y} = \frac{\int_0^{2\pi}\int_0^{1-\sin\theta} y \cdot r\,dr\,d\theta}{\int_0^{2\pi}\int_0^{1-\sin\theta} r\,dr\,d\theta} = \frac{\frac{-5\pi}{4}}{\frac{3\pi}{2}} = -\frac{5}{6}$$

2. Do #60 in §13.6.



(0,.5) - center of mass

(.385, .231) - population center

$$\bar{x} = \frac{10{,}000(-2) + 15{,}000(-3) + 15{,}000(2) + 20{,}000(2) + 5{,}000(4)}{10{,}000 + 14{,}000 + 15{,}000 + 20{,}000 + 5{,}000} = \frac{2{,}500}{65{,}000} = .385$$

$$\bar{y} = \frac{10{,}000(2) + 15{,}000(-2) + 15{,}000(-3) + 20{,}000(0) + 5{,}000(-4)}{10{,}000 + 15{,}000 + 15{,}000 + 20{,}000 + 5{,}000} = \frac{15{,}000}{65{,}000} = .231$$

3. A major city has a population varying linearly with distance from its center, with ½ million people per square km at its center, dropping to ¼ million people per square km at a distance of 5km from the center.
   a) What's the total population living within 5km of the city's center?

   b) Do more people live within 2km of the city's center, or between 2km and 5km?

   c) In case of an epidemic, at which points in the city are people living within 1km of at least 1 million people?

$$\delta(r) = \frac{1}{2} - \frac{1}{20}r$$

a) $\int_0^{2\pi}\int_0^{5}\left(\frac{1}{2} - \frac{1}{20}r\right) r\,dr\,d\theta = \frac{25\pi}{3}$

b) $\frac{26\pi}{15}$ vs. $\frac{33\pi}{5}$

c) Well, within 1km there are $\frac{7\pi}{15}$ million, so there are some at least.