Quiz 2 Calculus 3 11/16/15

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Let $\mathbf{F}(x,y) = \langle 5x^4y^2, 2x^5y \rangle$, and C be the line segment from (2,–1) to (1,3).
   Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.

$f(x,y) = x^5y^2$ is potential function.

$f_x(x,y) = 5x^4y^2$

$f_y(x,y) = 2x^5y$

So, we will use the Fun. Theorem.

$$f(1,3) - f(2,-1) = (1)^5(3)^2 - (2)^5(-1)^2$$

$$= 9 - 32$$

$$= -23$$

Good

~~$2x^2 - x + \frac{y^2}{2} - x^2 y$~~

2. Let $\mathbf{F}(x,y) = \langle 4x - 1, y - x^2 \rangle$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for C the second-quadrant portion of a circle with radius 3 centered at the origin, traversed counterclockwise.

I. $\vec{r}(t) = \langle 3\cos t, 3\sin t \rangle \quad \frac{\pi}{2} \le t \le \pi$

II. $\vec{F}(\vec{r}(t)) = \langle 12\cos t - 1, 3\sin t - 9\cos^2 t \rangle$

III. $\vec{r}\,'(t) = \langle -3\sin t, 3\cos t \rangle$

IV. $\int_{\frac{\pi}{2}}^{\pi} \langle 12\cos t - 1, 3\sin t - 9\cos^2 t \rangle \cdot \langle -3\sin t, 3\cos t \rangle \, dt$

V $\int_{\frac{\pi}{2}}^{\pi} -36\sin t\cos t + 3\sin t + 9\sin t\cos t - 27\cos^3 t$

$\int_{\frac{\pi}{2}}^{\pi} 3\sin t - 27\sin t\cos t - 27\cos^3 t \, dt$

$3\int_{\frac{\pi}{2}}^{\pi} \sin t\,dt - 27\int_{\frac{\pi}{2}}^{\pi} \sin t\cos t\,dt - 27\int_{\frac{\pi}{2}}^{\pi} \cos^3 t\,dt$

$3\left[-\cos t\Big|_{\frac{\pi}{2}}^{\pi}\right] - 27\int_{\frac{\pi}{2}}^{\pi} u\,du - 27\int_{\frac{\pi}{2}}^{\pi} \cos t(1 - \sin^2 t)\,dt$

$-3[-1-0]$

$3 \quad -27\left[\frac{\sin^2 t}{2}\Big|_{\frac{\pi}{2}}^{\pi}\right] - 27\int_{\frac{\pi}{2}}^{\pi} \cos t\,dt + 27\int_{\frac{\pi}{2}}^{\pi} \cos t \sin^2 t\,dt$

$[0 - \tfrac{1}{2}]$

$3 + \frac{27}{2} \qquad -27\left[\sin t\Big|_{\frac{\pi}{2}}^{\pi}\right] + 27\int_{\frac{\pi}{2}}^{\pi} u^2\,du$

$[0 - 1]$

$3 + \frac{27}{2} + 27 - 9 \qquad + 27\left[\frac{\sin^3 t}{3}\Big|_{\frac{\pi}{2}}^{\pi}\right]$

$[0 - \tfrac{1}{3}] = -27/3 = 9$

$= \frac{69}{2}$

Wonderful!