Each problem is worth 10 points. For full credit provide complete justification for your answers.

Use the graph of $g(x)$ at the bottom of the page for problems 1 and 2:

1. Find the following limits:
    a) $\lim\limits_{x \to -3^-} g(x)$
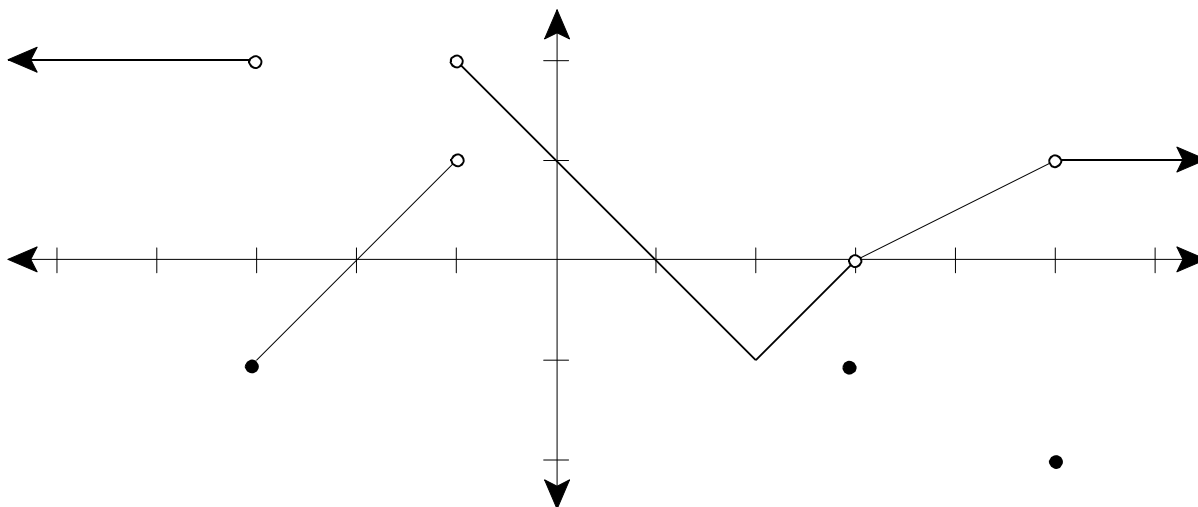
    b) $\lim\limits_{x \to -3^+} g(x)$

    c) $\lim\limits_{x \to -3} g(x)$

    d) $\lim\limits_{x \to 5^+} g(x)$

    e) $\lim\limits_{x \to 5^-} g(x)$

    f) $\lim\limits_{x \to 5} g(x)$

2. For which values of x does the function fail to be continuous?

3. Evaluate $\displaystyle\lim_{x\to 3}\frac{x^2-x-6}{x^2-9}$

4. Let $f(x)=\begin{cases}\sqrt{-x} & \text{if } x<0\\ 3-x & \text{if } 0\le x<3\\ (x-3)^2 & \text{if } x>3\end{cases}$. Evaluate each limit, if it exists:

   a) $\displaystyle\lim_{x\to 0^-}f(x)=$

   b) $\displaystyle\lim_{x\to 0^+}f(x)=$

   c) $\displaystyle\lim_{x\to 0}f(x)=$

   d) $\displaystyle\lim_{x\to 3^-}f(x)=$

   e) $\displaystyle\lim_{x\to 3^+}f(x)=$

   f) $\displaystyle\lim_{x\to 3}f(x)=$

5. If a mango is thrown straight up into the air with an initial velocity of 90 ft/s, its height in feet after t seconds is given by $y = 90t - 16t^2$. Find the average velocity for the time period beginning when $t = 1$ and lasting

   a) 0.5 seconds

   b) 0.1 seconds

   c) 0.01 seconds

6. a) Evaluate $\displaystyle\lim_{x\to5^-}\frac{2x^2+3}{(x-5)(x+2)}$.

b) Evaluate $\displaystyle\lim_{x\to\infty}\frac{2x^2+3}{(x-5)(x+2)}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. Our Calc class makes this big deal about stuff being numeric sometimes, which I thought was pretty much always how math was, right? But there was this one question on our test prep stuff they gave us, like that you want a limit for close to 0, right? And the function was, like sin π/x, right? And if you plugged in 0.1, and then you plugged in 0.01, and then you plugged in 0.001, then every time you get 0, right? But so they said it like you were supposed to say how you know the limit isn't really 0, but I say, three times in a row can't be an accident, right?"

Help Biff by explaining as clearly as you can why $\lim_{x \to 0} \sin \dfrac{\pi}{x}$ is *not* 0, despite the numerical evidence he mentions.

8. Evaluate $\lim\limits_{h \to 0} \dfrac{(5+h)^2 - 5^2}{h}$.

9. Is there a number that is exactly 1 more than its cube? How can you be sure?

10. Consider a continuous function with the following properties:

$$\lim_{x \to \infty} f(x) = 1 \qquad \lim_{x \to -\infty} f(x) = 1 \qquad f(0) = 3$$

a) Sketch a graph of a function having the properties listed above.

b) Find a formula for such a function.

Extra Credit (5 points possible):

Evaluate $\lim_{x \to \infty} \sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1}$.