Exam 1a Calc 1 9/16/2016

Each problem is worth 10 points. For full credit provide complete justification for your answers.

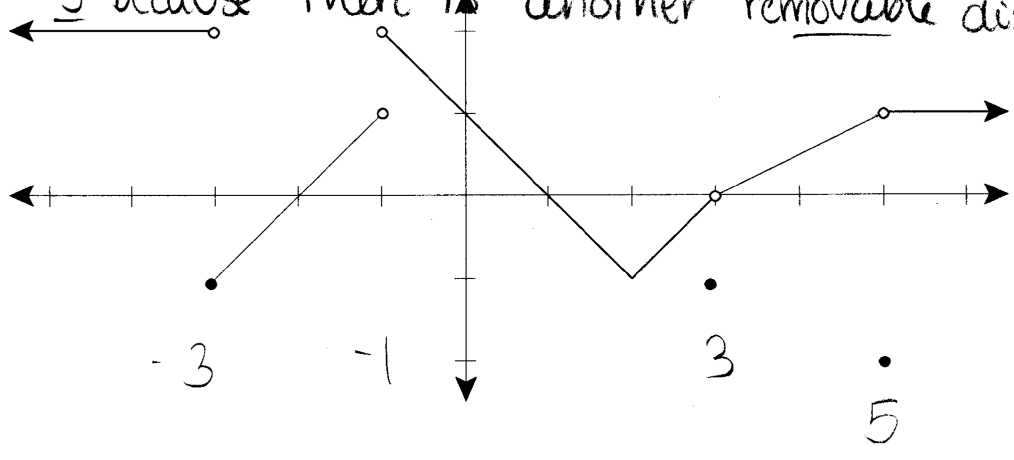
Use the graph of $g(x)$ at the bottom of the page for problems 1 and 2:

1. Find the following limits:
   a) $\lim_{x \to -3^-} g(x) = \underline{2}$
   b) $\lim_{x \to -3^+} g(x) = \underline{-1}$
   c) $\lim_{x \to -3} g(x) = \underline{DNE}$ $\lim_{x \to -3^+} \neq \lim_{x \to -3^-}$
   d) $\lim_{x \to 5^+} g(x) = \underline{1}$ Great
   e) $\lim_{x \to 5^-} g(x) = \underline{1}$
   f) $\lim_{x \to 5} g(x) = \underline{1}$

2. For which values of x does the function fail to be continuous?

It is not continuous at $\underline{-3}$ because $\lim_{x \to -3^-} g(x) \neq \lim_{x \to -3^+} g(x)$ at $\underline{-1}$ because $\lim_{x \to -1^-} g(x) \neq \lim_{x \to -1^+} g(x)$, at $\underline{3}$ because there is a removable discontinuity, and at $\underline{5}$ because there is another removable discontinuity

Excellent!



3. Evaluate $\lim\limits_{x\to 3}\dfrac{x^2-x-6}{x^2-9}$

$$= \lim_{x\to 3}\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)}$$

$$= \lim_{x\to 3}\frac{x+2}{x+3}$$

$$= \frac{3+2}{3+3}$$

$$= \frac{5}{6}.$$

Excellent!

4. Let $f(x)=\begin{cases} \sqrt{-x} & \text{if } x<0 \\ 3-x & \text{if } 0\le x<3 \\ (x-3)^2 & \text{if } x>3 \end{cases}$. Evaluate each limit, if it exists:

a) $\lim\limits_{x\to 0^-} f(x) = \underline{0}$ $\quad \sqrt{0} = 0$

b) $\lim\limits_{x\to 0^+} f(x) = \underline{3}$ $\quad 3-0=3$

c) $\lim\limits_{x\to 0} f(x) =$ DNE because $\lim\limits_{x\to 0^-}$ and $\lim\limits_{x\to 0^+}$ are not equal

d) $\lim\limits_{x\to 3^-} f(x) = \underline{0}$ $\quad 3-3=0$

Excellent!

e) $\lim\limits_{x\to 3^+} f(x) = \underline{0}$ $\quad (3-3)^2 = 0$

f) $\lim\limits_{x\to 3} f(x) = \underline{0}$ because $\lim\limits_{x\to 3^-}$ and $\lim\limits_{x\to 3^+}$ are equal

5. If a mango is thrown straight up into the air with an initial velocity of 90 ft/s, its height in feet after t seconds is given by $y = 90t - 16t^2$. Find the average velocity for the time period beginning when $t = 1$ and lasting

a) 0.5 seconds 50 ft/s

b) 0.1 seconds 56.4 ft/s

c) 0.01 seconds 57.84 ft/s

t	h(t)
1	74
1.01	74.5784
1.1	79.64
1.5	99

Nice!

$$\frac{99-74}{1.5-1} = \frac{25}{0.5} = 50\text{ ft/s}$$

$$\frac{79.64-74}{1.1-1} = \frac{5.64}{0.1} = 56.4\text{ ft/s}$$

$$\frac{74.5784-74}{1.01-1} = \frac{0.5784}{0.01} = 57.84\text{ ft/s}$$

6. a) Evaluate $\lim_{x\to 5^-} \frac{2x^2+3}{(x-5)(x+2)}$. $= \lim_{x\to 5^-} \frac{\overset{+}{(2x^2+3)}}{(\text{approaching } \overset{-}{0})(\overset{+}{x+2})}$

$(-)\cdot(+)\cdot(+) = -$

thus

$\lim_{x\to 5^-} \frac{2x^2+3}{(x-5)(x+2)} = -\infty$

Excellent.

b) Evaluate $\lim_{x\to\infty} \frac{2x^2+3}{(x-5)(x+2)}$. $= \lim_{x\to\infty} \frac{2x^2+3}{x^2-3x-10} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} =$

$\lim_{x\to\infty} \frac{2+\cancel{\frac{3}{x^2}}^{0}}{1-\cancel{\frac{3}{x}}^{0}-\cancel{\frac{10}{x^2}}^{0}} = 2$

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. Our Calc class makes this big deal about stuff being numeric sometimes, which I thought was pretty much always how math was, right? But there was this one question on our test prep stuff they gave us, like that you want a limit for close to 0, right? And the function was, like sin π/x, right? And if you plugged in 0.1, and then you plugged in 0.01, and then you plugged in 0.001, then every time you get 0, right? But so they said it like you were supposed to say how you know the limit isn't really 0, but I say, three times in a row can't be an accident, right?"

Help Biff by explaining as clearly as you can why $\lim_{x\to 0} \sin\frac{\pi}{x}$ is *not* 0, despite the numerical evidence he mentions.

Plugging in 0.1, 0.01, and 0.0001 only give you values of 0 because the graph oscilates between -1 and 1. You happened to bais the values you chose. If you chose values like 0.0365 you get -0.948362 or 0.0549 you get 0.625086. You have to be careful of the values you choose when you are working with sin or cos because the bounce between one and negative one. I would also graph the function to show Biff a visual of why it is not 0.

Excellent explanation.

8. Evaluate $\lim_{h \to 0} \frac{(5+h)^2 - 5^2}{h}$.

$$= \lim_{h \to 0} \frac{\cancel{25} + 10h + h^2 - \cancel{25}}{h}$$

$$= \lim_{h \to 0} \frac{10h + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(10 + h)}{\cancel{h}}$$

well done

$$= \lim_{h \to 0} 10 + h$$

$$= 10 + 0$$

$$= \underline{10}$$

9. Is there a number that is exactly 1 more than its cube? How can you be sure?

Well, call it x, so we'd have

$$x = 1 + x^3$$

I'm going to rearrange that to

$$0 = 1 + x^3 - x$$

And then think about it as a function, named f, so

$$f(x) = 1 + x^3 - x$$

Now notice that when I put 0 in,

$$f(0) = 1 + (0)^3 - (0) = 1$$

Whereas when I put -2 in,

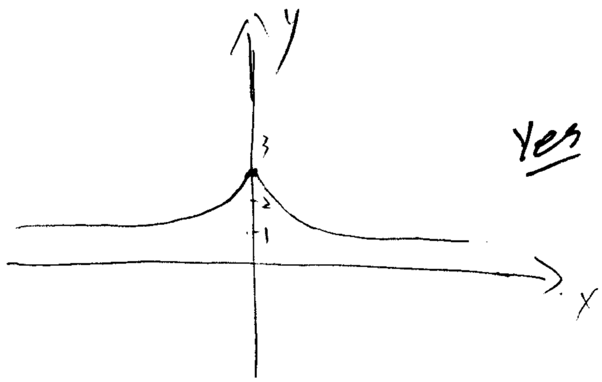
$$f(-2) = 1 + (-2)^3 - (-2) = -5$$

So since 0 is a height in between 1 and -5, and this function is continuous (since it's a polynomial), there must be an input between 0 and -2 whose output is 0, which means there <u>is</u> an x that meets this requirement, by the Intermediate Value Theorem.

10. Consider a continuous function with the following properties:

$$\lim_{x\to\infty} f(x) = 1 \qquad \lim_{x\to-\infty} f(x) = 1 \qquad f(0) = 3$$

a) Sketch a graph of a function having the properties listed above.



Yes

b) Find a formula for such a function.

$f(x) = \dfrac{x^2+3}{x^2+1}$

$\lim\limits_{x\to\infty} \dfrac{x^2+3}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{1+\frac{3}{x^2}}{1+\frac{1}{x^2}}$

$= 1$

$\lim\limits_{x\to-\infty} \dfrac{x^2+3}{x^2+1} = \lim\limits_{x\to-\infty} \dfrac{1+\frac{3}{x^2}}{1+\frac{1}{x^2}}$

$= 1$

$f(0) = \dfrac{0+3}{0+1}$

$= 3.$

$\therefore f(x) = \dfrac{x^2+3}{x^2+1}$

Excellent