

**Exam 2    Calc 1    10/7/2016**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the derivative of a function  $f(x)$ .

2. a) What is the derivative of  $f(x) = x^5$ ?

b) What is the derivative of  $f(x) = \frac{1}{x^2}$  ?

c) What is the derivative of  $f(x) = \sin x$ ?

d) What is the derivative of  $f(x) = \cos x$ ?

e) What is the derivative of  $f(x) = \tan x$ ?

3. A table of values for  $f$ ,  $g$ ,  $f'$ , and  $g'$  is given below.

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	-16	2	4	10
2	6	9	2	4
3	8	2	-3	7

a) If  $h(x) = f(x) \cdot g(x)$ , what is  $h'(2)$  and why?

b) If  $h(x) = f(x) / g(x)$ , what is  $h'(1)$  and why?

c) If  $h(x) = f(g(x))$ , what is  $h'(3)$  and why?

4. State and prove the Difference Rule for derivatives.

5. Use the definition of the derivative to show that the derivative of  $f(x) = \sqrt{x}$  is

$$f'(x) = \frac{1}{2\sqrt{x}}.$$

6. State and prove the Product Rule for derivatives. Make it clear how you use any assumptions.

7. Buffy is a calculus student at Enormous State University, and she's having some trouble. Buffy says "*Ohmygod*. This Calculus stuff is so hard! I swear, ninety percent of the time I just have no idea what the professor is talking about at all. Yesterday she was going on and on about what happens if you do the derivative of  $x$  times  $x^{n-1}$ . She was saying it over and over, but she didn't write it on the board, and my notes make no sense at all! What could the point have been?"

Help Buffy by explaining as clearly as possible what happens when you do the derivative of her professor's function, and what good that is.

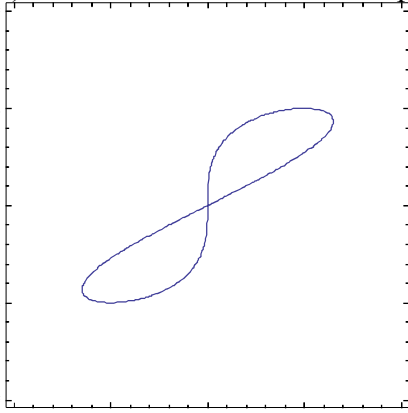
8. a) Write a linearization for  $f(x) = x^4$  at  $x = 2$ .

b) Use the linearization from part a to approximate  $(1.999)^4$ .

9. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 2:00 PM?



10. a) Find the derivative with respect to  $x$  of the curve with equation  $x^2 - 2xy + y^4 = 0$



b) Find the points on the curve from part a where the tangent line is vertical.

Extra Credit (5 points possible): State and prove the Quotient Rule for derivatives. Make it clear how you use any assumptions.