Exam 4 Calc 1 11/18/2016

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find all critical numbers of $f(x) = 2x^3 - 3x^2 - 36x$.

$f(x) = 2x^3 - 3x^2 - 36x$

$f'(x) = 6x^2 - 6x - 36$ take derivative

$0 = 6x^2 - 6x - 36$ set derivative = to 0

$0 = 6(x^2 - x - 6)$ factor out 6

$0 = x^2 - x - 6$ divide both sides by 6

$0 = (x-3)(x+2)$ factor

Critical numbers 3 + -2 solve for x

Great!

2. Find all inflection points of $f(x) = 2x^3 - 3x^2 - 36x$.

$f'(x) = 6x^2 - 6x - 36$

$f''(x) = 12x - 6$

$12x - 6 = 0$

$12x = 6$

$x = 1/2$

Points of inflection @ 1/2

Good

3. The function $f(t) = \dfrac{1}{0.2 + e^{-t}}$ is being used to model a learning curve for students in a study. What is the largest interval on which f is concave down? [Hint: $f'(t) = \dfrac{25e^t}{\left(e^t + 5\right)^2}$ and $f''(t) = \dfrac{-25e^t(e^t - 5)}{\left(e^t + 5\right)^3}$]

$0 = -25e^t(e^t - 5)$

$0 = e^t \qquad 0 = e^t - 5$

$\nexists \qquad \ln 5 = \ln e^t$

$\ln 5 = t$

$(-\infty, \ln 5)(\ln 5, \infty)$

$+ \qquad -$

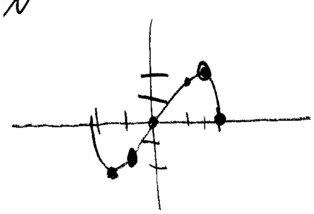
Great

from $(\ln 5, \infty)$

10

4. Find the absolute minimum and maximum values of $f(t) = t\sqrt{4 - t^2}$ on $[-1,2]$.

$(-1, -\sqrt{3})$
$(0, 0)$
$(1, \sqrt{3})$
$(2, 0)$



$f(t) = t \cdot (4 - t^2)^{1/2}$

$f'(t) = t \cdot \cancel{\tfrac{1}{2}}(4 - t^2)^{-1/2} \cdot -\cancel{2}t + (4 - t^2)^{1/2} \cdot 1$

$f'(t) = (4 - t^2)^{-1/2}\left[-t^2 + (4 - t^2)\right]$

$0 = \dfrac{-2t^2 + 4}{\sqrt{4 - t^2}}$

$-2t^2 + 4 = 0$

$2t^2 = 4$

$\sqrt{t^2} = \sqrt{2}$

$t = \pm\sqrt{2}$

absolute minimum $(-1, -\sqrt{3})$

absolute maximum $(\sqrt{2}, 2)$

Excellent!

5. Use one iteration of Newton's Method (so find x_1) with an initial approximation of $x_0 = 2$ to approximate a solution to $0 = 2x^3 - 5x^2 + 2x - 7$.

$f'(x) = 6x^2 - 10x + 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 2 - \frac{f(2)}{f'(2)}$

$x_1 = 2 - \frac{-7}{6}$

$x_1 = 2 + 7/6$

Great

$\boxed{x_1 = \frac{19}{6}}$

6. If $f''(x) = -2 + 12x - 12x^2$, $f(0) = 6$, and $f'(0) = 12$, what is $f(x)$?

$f'(x) = -2x + 6x^2 - 4x^3 + C \rightarrow 12 = -2(0) + 6(0)^2 - 4(0)^3 + C$

$C = 12$

$f'(x) = -2x + 6x^2 - 4x^3 + 12$

$f(x) = -x^2 + 2x^3 - x^4 + 12x + D$

$6 = -(0)^2 + 2(0)^3 - (0)^4 + 12(0) + D$

$D = 6$

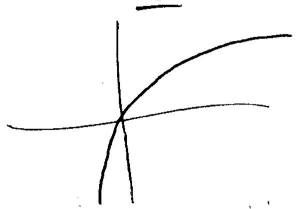
$\boxed{f(x) = -x^2 + 2x^3 - x^4 + 12x + 6}$

Excellent!

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this calculus stuff is tough. I was understanding the derivative stuff pretty well, with the increasing and decreasing and stuff, right? But then the professor started talking about the second derivative, so like if increasing is increasing or decreasing, I guess? But I don't get it, and he was talking about, like, caves or something? Like cave up and cave down? But his accent is really tough, and I don't think I get it."

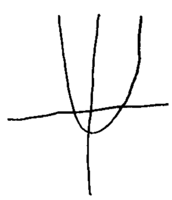
Explain clearly to Biff what the second derivative tells us about the shape of a graph.

The second derivative tells us the shape of the graph by telling us if the slope is increasing or decreasing. A negative second derivative means the graph has a concave down shape, or a slope that is decreasing. Graphs like that can look like:



or

Likewise, a positive second derivative means a graph is concave up word, like:



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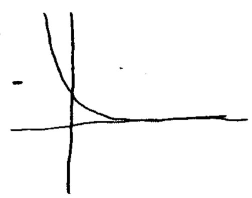
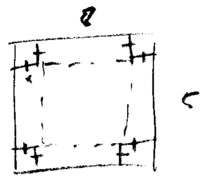


This because the second derivative tells us how fast the slope changes. That, in turn, helps us picture the graph of a function more clearly!

Good.

✓ 8. [B&C] Squares with sides of length x are cut out of each corner of a rectangular piece of cardboard measuring 5 ft by 8 ft. The resulting piece of cardboard is then folded into a box without a lid. Find the volume of the largest box that can be formed in this way.



$V(x) = (8-2x)(5-2x)(x)$

$V(x) = (40 - 16x - 10x + 4x^2)(x)$

$V(x) = 40x - 16x^2 - 10x^2 + 4x^3$

$V(x) = 40x - 26x^2 + 4x^3$

$V'(x) = 40 - 52x + 12x^2 \rightarrow 0 = 40 - 52x + 12x^2$

$$\frac{52 \pm \sqrt{(-52^2 - 4(12)(40))}}{2(12)} = \frac{52 \pm \sqrt{(784)}}{24} = \frac{52 \pm 28}{24}$$

critical points:

$x = \frac{80}{24} = \frac{20}{6} = \frac{10}{3}$

$x = 1$

$V(\frac{10}{3}) = 40(\frac{10}{3}) - 26(\frac{10}{3})^2 + 4(\frac{10}{3})^3$

$\frac{10}{3} = -7.4074074$

not the max

$V(1) = 40(\cancel{1}) - 26(\cancel{1})^2 - 4(\cancel{1})^3$

$V = 40 - 26 + 4$

$V = 40 - 22$

$\boxed{V = 18}$

Great

the physics student in me [illegible]

Real data. Obviously we could do it in SI, but the moral (electric vehicles have low optimal speeds) is lost when people don't have a feel for the units. Your point is still generally great.

9. A newly developed electric vehicle has energy consumption (measured in Watt-hours per mile driven) given by $E(v) = 0.2v^2 + 1000/v$, where v is the velocity of the vehicle (in miles per hour). At what velocity (exactly) will the car's energy consumption for a trip be lowest?

$E(v) = 0.2v^2 + \frac{1000}{v}$

$E(v) = 0.2v^2 + 1000v^{-1}$

$E'(v) = .4v - 1000v^{-2}$

$0 = .4v - 1000v^{-2}$

$0 = .4v - \frac{1000}{v^2}$

$\frac{1000}{v^2} = .4v$

$1000 = .4v^3$

$2500 = v^3$

$13.57 = v$

Good

$\boxed{\sqrt[3]{2500} \text{ or } \approx 13.572 \text{ mph}}$

10. A can of pumpkin pi filling is being designed to hold a volume of 200π cm^3. What radius and height will minimize the surface area (bottom, top, and sides) of the can?

[Tips: For a circle, Area = πr^2 and Circumference = $2\pi r$. For a cylinder, Volume = πr^2.]

$200\cancel{\pi} = \cancel{\pi} r^2 h$

$h = \frac{200}{r^2}$

$$\boxed{h = \frac{200}{(100)^{2/3}} \text{ cm}}$$

$A(r) = 2\pi r^2 + 2\pi r(h)$

$A(r) = 2\pi r^2 + 2\pi \cancel{r} \frac{200}{r^{\cancel{2}}}$

$A'(r) = 4\pi r - \frac{400\pi}{r^2}$

$0 = 4\pi r - \frac{400\pi}{r^2}$

$\overset{100}{\cancel{400}}\cancel{\pi} = \cancel{4}\cancel{\pi} r^3$

$$\boxed{r = \sqrt[3]{100} \text{ cm}}$$

Excellent!